THE
Countrey-Survey-Book:
OR
LAND-METERS
VADE-MECUM.

WHEREIN

The PRINCIPLES and *practical* RULES for *Surveying of Land*, are so plainly (though briefly) delivered, that any one of ordinary parts (understanding how to *add*, *subtract*, *multiply* and *divide*,) may by the help of this small *Treatise* alone and a few cheap *Instruments* easy to be procured, *Measure* a parcel of Land, and with judgment and expedition *Plot it*, and give up the Content thereof.

WITH

An *APPENDIX*, containing *Twelve Problems* touching **Compound Interest** and **Annuities**; and a *Method* to *Contract* the work of **Fellowship** and **Alligation** *Alternate*, very considerably in many Cases.

Illustrated with Copper Plates.

By *ADAM MARTINDALE*,
A Friend to Mathematical Learning.

Frustrà fit per plura quod fieri potest per pauciora.

London, Printed by *A. G.* and *F. P.* for *R. Clavel*, at the *Peacock* in St. *Pauls* Church-yard, 1682.

Thom. Parnell R 6


BIBLIOTHECA BODLEIANA

TO THE

Right Honourable

THE LORD

DELAMER,

Baron of *Dunham-Maſſie*, &c.

MY LORD,

THIS *ſmall Tract comes to Your Lordſhip, not as to a Patron to protect its Errors (if any ſuch there be) but as to a critical noble Friend, that will be ſure faithfully to tell the Author of them. Which favour, together with a chearful acceptance of this poor preſent, he humbly hopes for, becauſe of its Relation, being writ at* Dunham, *by Your humble Servant,*

Servant, who beſides his domeſtick dependance, cannot forbear without ingratitude to tell the World that Your Lordſhip's kindneſs hath very much encouraged and aſſiſted him in Mathematical Studies, not onely by a free Communication of many a choice notion both vivâ voce, *and by the loan of Manuſcripts. But alſo by a Conſiderable number of excellent Books and coſtly Inſtruments bountifully beſtowed upon him: Who wanting other ways to expreſs his many ſingular Obligations, and deep ſence thereof; humbly offers this Punie Treatiſe for Your Lordſhips diverſion at ſpare hours; and is ambitious to write himſelf,*

MY LORD,

Oct. 26 1681.

An humble and faithful
(though unworthy)
Servant to all Your
Lordſhip's Noble Family

A. M.

Mr. *COLLINS* TO THE READER.

Courteous Reader,

THE Learned Mr. *Adam Martindale* formerly Writ two excellent Almanacks, called Country Almanacks, which were Printed, and esteemed by several Members of the Royal Society very useful, especially for Country Affairs, but meeting with some Discouragements from such as knew not how to judge of the Authors worth, he gave over that undertaking, contrary to the desires of many Ingenious Men.

And having since Writ a little Treatise of Survey, in which he hath had experience, as well as Theory, and being willing to have the Approbation or Dislike of others, that it might either be Printed or Stifled, imparted the same to some of the Members

Members of the aforeſaid Illuſtrious Body.

Upon the Peruſal of whom and diverſe Experienced Artiſts who make a Livelyhood of it, I find it well Approved (as Clear and Conciſe) only the latter were ſorry ſo much was diſcovered, as detrimental to their Practiſe, particularly about ſetting off the outjettings, where its inconvenient intirely to meaſure them.

The Book is ſmall, the price (though not the worth) mean, no ſmall Encouragements to young Students and the Vulgar, for whom it was chiefly intended: By which, that they may reap Benefit, is the hearty deſire of (a Well-willer to the Author and them)

John Collins.

The PREFACE.

READER,

THe Title ſufficiently informs thee in general of my Deſigns: But I confeſs I owe thee a more particular Account, not only of that, but ſeveral other things which I ſhall briefly give thee.

1. I have obſerved that the Countrey aboundeth with ſuch, as by their Inclination and Intereſt are prevailed with to take pains in meaſuring Land, that for want of better Inſtruction uſe ill-divided Chains and tedious Methods of Computation, which makes their work intolerable troubleſom, if exactneſs be required. And ſome for want of skill in the Fundamentals of Geometry, have imbibed prodigiouſly falſe Principles, as this for one, *viz.* That *the Content of any Cloſe, of what Figure ſoever, may be found by Squaring a quarter of the Perimeter.* Mathematical-Schools, where better things might be learned, are very rare, and an able Artiſt to inſtruct one in private is hard, and chargeable to be procured. Excellent Books indeed there are in our *Engliſh* Tongue, Written by our Famous *Rathborn*, *Wing*, *Leybourne*, and *Holwell*, to which may be added Induſtrious Mr. *Atwells* Treatiſe, and ſome part of Capt. *Sturmy*'s: But thoſe I rather eſteem fit to be read by an able Artiſt (towards his perfecting) than by a new-beginner, for in the beſt of thoſe Books he will find the moſt uſeful and plain Rules ſo intermixed with others that are leſs neceſſary, and more intricate, (though very excellent for their proper ends) and ſo many Curioſities touching Trigonometry, Tranſmutation of Figures, *&c.* which his

his buſineſs never calls for, that for want of judgment to pick out that which fits his preſent purpoſe, and to ſtudy higher Speculations afterwards, he is apt to be confounded and diſcouraged; whereby it accidentally comes to paſs that plenty, makes him poor.

Beſides, three of theſe ſix Books are in Folio, another in Quarto, and the other two (though Octavos) too large for ordinary carrying in a Pocket, and in that regard not ſo convenient for one that hath much occaſion to be out of his own Houſe; to ſay nothing of their Price, which to ſome poor Youths is not the leaſt diſcouragement.

I have therefore made my Book ſo little, that the Price can neither much empty the Pocket, nor the Bulk overfill it: And yet ſo plain, that I doubt not to be underſtood by very ordinary Capacities. My Method is fitted to my Deſign: Beginning with the Principles of the Art, and ſo proceeding *gradatim* till I have ſhewed how all ordinary Figures may be Meaſured, Protracted, and Caſt up, without any other Inſtrument of charge but Chains, Compaſſes and Scales. Afterwards for ſuch as deſire higher Attainments, I have endeavoured to ſpeak ſo fully (in a little compaſs) of the *Plain Table*, and given ſuch hints, applicable not only to it, but alſo to the *Peractor*, *Theodolite*, and *Semicircle*, as that an Ingenious Perſon may make great uſe of them.

But as touching the Doctrine of Triangles, and Tranſmutation of Figures into others equipollent, with the large Tables (referring to the former) of Logarithms of Numbers, Sines, and Tangents, I thought it improper to cumber this ſmall Manual, or the unlearned Reader with them, having (as I humbly

humbly hope) ſufficiently inform'd him how to find all his Sides and Angles by Inſtruments, and alſo the Content of any Figure without ſuch tranſmutation, reſerving ſuch Curioſities and many others touching Drawing and Painting of Maps; Meaſuring of Wayes, and Rivers, *&c.* to a Second Part, which I may perhaps hereafter Publiſh upon due encouragement, but if I do not, the Curious may find themſelves good ſtore of work in the Authors even now quoted.

2. It may ſeem ſtrange to ſome, that in referring to the Figures, I ſometimes uſe Words ſeeming to imply that the Figure I ſpeak of is in that very Page. and ſo it was in my Copy, but the Printer and Gravers have otherwiſe contrived them for convenience in Copper Cuts by themſelves. And to give them their due they are generally done with great accuracy, and none of them having any ſuch errour as is like to beget trouble or miſtake to the Reader, ſaving only that fig. 19 hath D inſtead of O at the Center, and the Line O L in the Margin of *Fig.* 14. ſhould be of the length from L to the uppermoſt o in the Scale, and the Figures on the ſide ſhould be made 1 leſs than they are, *viz.* 2 ſhould be 1, 3 made 2, *&c.*

And laſtly, as to the *Errata*, though I have not been ſo anxiouſly careful as to Correct every *literal miſtake*, I have very diligently peruſed all from p. 1 to p. 224 incluſive, and hope I have ſufficiently reſtored the Sence to the places wronged, when thou haſt done them right by thy Pen according to the Directions of the Table following next after the *Contents*, and that you continue the Line in the Margin of p. 34. to the length of the Line O L in fig. 14.

THE

THE CONTENTS OF THE CHAPTERS.

ERRATA

Pag.	Li.	Read
3	16	Trilaterals.
4	3	Geodetes.
5	29	Eneagon.
10	6	be ſo long as the other two.
16	6	Centers at Right-Angles.
28	1	forefinger.
36	20	Poles or Rods.
47	15	fourth Diagonal and the ſixth ſide.
49	28	as in this figure is *A B C*.
56	23	21; *and line* 24 *read* 22 $\frac{1}{2}$.
58	11	28.
69	5	ſides.
74	25	*F G*.
77	27	138562, *and line* 28 *and* 33 *read* 242030.
79	33	triangulate.
80	9	former.
82	24	fitted.
86	2	Stationary diſtances.
90	15	Chart, *or* Card.
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104	19	let.
112	12	in it at pleaſure.
118	12	ſo round.
121	11	of Boats.
194	11	*dele* 3625.
200, 201, 202		31.188 *or* 31 l. 3 s. 9 d. 1 q. *every where inſtead of* 51.182, *or* 31 l. 3 s. 7 d. 3 q.
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THE *Country Survey-Book*: OR LAND-METER's *VADE-MECUM.*

CHAP. I.

Of Geometrical Definitions, Diviſions, and Remarks.

I. A *Point* is that which hath no parts, either of longitude or latitude, but is indiviſible, ordinarily expreſſed with a ſmall prick, like a period at the end of a ſentence.

II. A *Line* hath length, but no bredth nor depth, whoſe limits or extremities are Points. This is either *right* or *crooked*:

III. A *right Line* lies ſtraight, and equal between its extreme points, being the ſhorteſt extenſion between them; the *crooked* or *circular* not ſo.

IV. A *Superficies* hath length and bredth, but no depth; of this Lines are the limits.

V. A *plain Superficies* is that which lieth equally (or evenly) between its Lines.

VI. An *Angle* is the Meeting of two Lines in one Point, ſo as not to make one ſtraight Line, and if drawn on paſt that Point, they will interſect or croſs one another. This in vulgar *Engliſh* may be called a *Corner*; of which there be two ſorts, one *right*, the other *oblique*.

VII. A *right Angle* is that which is made by two right Lines, croſſing or touching one another perpendicularly, (or ſquarely) like an ordinary Croſs, or Carpenters Square.

VIII. An *oblique Angle* is that which is either greater or leſs than a right Angle, and this is of two ſorts, *obtuſe* and *acute*.

IX. An *obtuſe Angle* is greater than a right Angle, like the left and right Corners of a Roman X.

X. An *acute Angle* is leſs than a right Angle, like the higheſt and loweſt Corners of the ſame Letter.

XI. A *Figure* is that which is comprehended under one Line or many: Of this there are two kinds, a *Circle* and a *right-lined Figure*.

XII. A *Circle* is a perfectly round Figure, ſuch as is drawn with a pair of Compaſſes, the one Foot being turned round in a Point, and the other wheeled about it. The Point in the preciſe

ciſe middle is called the *Center*; the round Line, the *Circumference* or *Periphery*; a Line going through the Center, and dividing the Circle into two equal parts, is called the *Diameter*; half of that Line is a *Semidiameter*, or *Radius*; half the Circle is ſtiled a *Semicircle*; the quarter, a *Quadrant*; any portion of it, cut off by a right Line not touching the Center, is called a *Segment*.

XIII. *Right-lined Figures* are ſuch as are limitted by three right Lines or more, and are either *Triangles*, or *Triangulate*, that is, ſuch as are compounded of, and reſolvable into Triangles.

XIV. *Triangles* are Figures comprehended under three right Lines, and (as *Ramus* thinks, for a Reaſon that he gives, *lib. 6. pr. 6.*) might be better called *Trilaterats*; but the name *Triangle* from the Number of Angles hath obtained.

Alſo from the Nature and Quantity of their Angles theſe Triangles are diſtinguiſhed into three ſorts: 1. *Rectangled*, having one right Angle; 2. *Obtuſe-angled*, having one obtuſe Angle; and 3. *Acute-angled*; having all acute Angles; for no Triangle can have more right or obtuſe Angles than one, becauſe by an old Rule (eaſie to be demonſtrated) no Triangle upon a plain Superficies can conſiſt of three greater Angles than ſuch, as being jointly taken, are equal to two Right.

Theſe three ſorts of Triangles may, according to the length and proportion of their ſides, be ſubdiſtinguiſhed into ſeven; for each of them may have either two equal Sides or none, and the Acute-angled may have all three Sides or Lines equal: To all which kinds, learned Men

give diſtinct *Greek* Names, which if mine *Engliſh* Reader have a mind to ſee, they are to be found in Mr. *Wing*'s *Geodætos Practicus, book* 1. *pag.* 6. for my preſent purpoſe the above-mentioned trimembred diſtinction will abundantly ſuffice; for be Triangles of what name or kind ſoever, they are all capable of being exactly meaſured by one plain Rule, as hereafter ſhall fully appear.

XV. *Triangulate Figures* are ſuch as have more Angles (and conſequently more Sides or Lines) than three; and theſe are either *quadrangular* or *multangular.*

XVI. *Quadrangular Figures* are ſuch as have four Angles (and as many Sides) and theſe are either *Parallellograms* or *Trapezia's.*

XVII. *Parallellograms* are Figures that are bounded with parallel Lines, that is, ſuch Lines as are every where of the ſame diſtance one from another, ſo as if they were infinitely extended they would never meet, like the upright Lines of the Roman H. Theſe *Parallellograms* are either *rectangular* or *obliquangular.*

XVIII. *Rectangular Parallellograms* are ſuch as have four right Angles, *viz.* the *Square* or *Quadrat*, and the *long Square*, otherwiſe called the *Oblong.*

XIX. The *Square* is that Figure that hath four right Angles, and four equal Sides, like any of the ſix Faces of a Die.

XX. The *long Square* hath alſo four right Angles, and the oppoſite Sides are equal, but the adjoyning Sides meeting at each Angle differ in length. Of this Figure is a well printed Page in a Book, and the ſuperficies of a well

cut

cut Sheet of Paper, or an ordinary Pane of Glaſs.

XXI. *Obliquangled Parallellograms* are ſuch as have oblique Angles, *viz.* two acute, and two obtuſe. Of theſe there are two kinds, the *Rhombus*, and the *Rhomboides*.

XXII. The *Rhombus* is a Figure that hath equal Sides, but no right Angles, (like the form of a Diamond on the Cards, or the moſt ordinary Cut of Glaſs in windows) whoſe oppoſite Angles are equal.

XXIII. The *Rhomboides* is (as it were) a defective *Rhombus*, for if from any Side of a *Rhombus* we cut off a part with a parallel Line, the Remainder will be a *Rhomboides*, which hath neither equal Sides nor Angles, but yet the oppoſite Sides and Angles are equal.

XXIV. The *Trapezium* is a Figure that is neither parallellogram, nor (conſequently) hath equal Sides or Angles, but is irregularly quadrangular, as if drawn at adventure. Of this ſhape moſt Fields prove, that ſeem to the Eye to be Squares or Oblongs.

XXV. *Multangular Figures* are ſuch as contain more Sides and Angles than four, and they are either *regular*, or *irregular*.

XXVI. *Regular Multangulars* take their Names from their Number of Angles, ſo a *Pentagon*, *Hexagon*, *Heptagon*, *Octogon*, *Encagon*, *Decagon*, ſignifie a multangular Figure of *five*, *ſix*, *ſeven*, *eight*, *nine*, *ten* Angles, and conſequently Sides.

XXVII. An *irregular Polygon*, or *multangular Figure*, is that which hath more Angles (and Sides) than four, the Sides (and Angles) being unequal to one another.

CHAP. II.

Of Geometrical Problems.

I. *To draw a Line parallel to another, at any Diſtance aſſigned.*

Fig. 1 OPen your Compaſſes to the Diſtance given, and chuſing two Points conveniently diſtant in the Line given, as here at A and B, deſcribe the Arches C and D, to whoſe convexity if you apply a Rule, the parallel Line is eaſily drawn.

II. *To raiſe a Perpendicular upon a Line given, or to croſs that Line at right Angles in a Point aſſigned.*

Fig. 2 Suppoſe the Point C in the Line AB were aſſigned for the Perpendicular; open the Compaſſes to a convenient diſtance, and mark out the two points E and F in the Line AB, then opening them ſomewhat wider, you may (by ſetting one Foot in E and F ſeverally) deſcribe the two Arches cutting one another at the point D, from which if you draw a Line to the point C, the work is done for the raiſing of a Perpendicular; but if you be to croſs the Line at right Angles, you may continue the Line from D through C at pleaſure.

But if the ſaid Line AB had been given to be divided in the preciſe middle, by another Line croſſing it at right Angles, the way were to ſet one

point

point of the Compaſſes in A and B ſeverally, and having deſcribed two Arches above the Line, interſecting one another as at D, do the like below the Line A B from the ſame points, and with the ſame extent of your Compaſſes, then through the ſeveral Interſections (a Rule being laid upon them) a Line may be drawn, cutting the given Line exactly in the middle at right Angles. *Fig.* 2

Note, That when one point of your Compaſſes ſtands in A, you may make both the Arches belonging to that Center above and below the Line, and then removing the Compaſſes to B, you may croſs them both.

III. *To raiſe a Perpendicular at the End of a Line.*

Let O R be the Line given, then to raiſe a Perpendicular at R, make five little equal diviſions, and taking four of them with your Compaſſes, ſet one Foot of your Compaſſes in R, and with the other deſcribe the Arch P P; then take the diſtance from R to 5, and placing one Foot in 3, with the other deſcribe the Arch B B, interſecting the former in the point S; then ſhall the Line S R (being drawn by a ſtraight Rule) be a Perpendicular to the Line O R. *Fig.* 3

IV. *To let fall a Perpendicular upon a given Line from any Point aſſigned.*

Open your Compaſſes ſo as one Foot being ſet in the aſſigned point the other may go clear over the Line given, and thereby deſcribe an Arch cutting the Line at two points; then ſhall the half

half distance between those two Points be the Point to which the Perpendicular may be drawn from the Point assigned. But if you think it too much pains to find the Point of half distance by trials, you may help your self by the second Problem: For if you describe two Arches intersecting one another on the farther side of the Line from the assigned Point, placing (to that purpose) the foot of your Compasses first in one of the Intersections of the given Line, and then in the other; you may by laying a Rule upon the assigned Point, and the Intersection of the two Arches, draw a Perpendicular from the said assigned Point, cutting the given Line at right Angles.

☞ *Note*, that all these Problems touching Perpendiculars, aim at no greater matter, than what may be performed in a Mechanical way with exactness enough (and much more neatly by avoiding unhandsom Pricks and Arches) by the help of a small Square exactly made, (or for want thereof a Plate Quadrant, or broad Rule, having a right Angle and true Sides) for if you apply one Leg of such a Square to any Line, so as the Angle of the Square may touch the end of the said Line, or any other Point where the Perpendicular is to be raised, you may by the other Leg draw the Perpendicular. In like sort to let fall a Perpendicular from a Point assigned, you need only to apply one Leg of the Square to the Line, so as the other may touch (at the same time) the assigned Point whence you may draw the Perpendicular, by that Leg that toucheth the Point.

If

If the Angle of your Square be a little blunt either through ill making or long uſing, you muſt allow for it when you apply it to the Point in a Line. And when you are drawing a Perpendicular, you muſt ſtop before you reach the given Line, and then by applying the Leg of your Square to that part of the Perpendicular already drawn, ſo as part of that Leg may paſs clearly over the given Line, you may draw the reſt of your Perpendicular as exactly as if the Angle had been true. The like courſe is to be taken when a Line is to be croſſed by another drawn quite through it at right Angles.

V. *An Angle being given, to make another equal to it.*

The Angle X A D being given, and a Line drawn at pleaſure as is the loweſt from the Point E, open your Compaſſes to any convenient diſtance, and ſetting one foot in A deſcribe the Arch B C. Then with the ſame extent ſetting one foot in E, with the other deſcribe the Arch G H, long enough to equal or exceed the other. Then taking the diſtance B C between the points of your Compaſſes, ſet one in G, and with the other mark the point H in the Arch G H, through which Point H a Line being drawn from the Point E, will make an Angle with the Line E G equal to the Angle given. *Fig.* 4

Note, when we ſpeak of the quantity of Angles, their equality, or inequality, we never regard the length of the Lines; for if you extend or contract them at pleaſure, the Angle is ſtill

ſtill the ſame. But that is the greateſt Angle whoſe Lines are fartheſt diſtant from one another, at the ſame diſtance from the Angular Point, or the place where its Lines meet.

VI. *Any three Lines being given (equal or unequal) ſo as no one of them be longer than the other two joyned together, to make a Triangle of them.*

Fig. 5 The Lines A, B, C, being given, ſet the Line A from D to E; then with your Compaſſes take the length of the Line B, and ſetting one Foot in D deſcribe the Arch P O. This being done, take with your Compaſſes the length of the Line C, and ſetting one foot in E with the other croſs the former Arch at F, from which Interſection drawing Lines by a Rule to D and E, the Triangle is finiſhed.

☞ *Note*, that if all the Sides, or two of them, be equal, the method is the ſame; but the labour leſs, becauſe we need not to take the ſame length twice over with the Compaſſes.

VII. *To find the Perpendicular of a Triangle, in order to the meaſuring of it.*

Fig. 6 Let the Line A B be accounted the Baſe, and from the Angle C let fall a Perpendicular as was taught, *Probl.* 4. upon that line at D, which is ready for taking off with Compaſſes and meaſuring on a Scale, of which hereafter in the Chapters of meaſuring the Content of Figures.

But if we have no occaſion to draw the Perpendicular, but only to know the length of it,

(as

(as it moſt frequently falls out in meaſuring) no more is needful but to ſet one foot of the Compaſſes in the Angular Point C, and extend the other to the Baſe A B, ſo as it may touch it, but not go beyond it; then have we the Perpendicular between the Points of the Compaſſes.

VIII. *One Side being given, how to make a Square.*

The Line C D being given, raiſe a Perpendicular at C of the length (at the leaſt) of the given Line; then taking the Line C D between the feet of your Compaſſes, ſet it upon the Perpendicular from the Angular Point C to A: With the ſame diſtance ſetting one foot in D deſcribe the Arch O P. Laſtly, with the ſame diſtance (or extent) ſet one foot in A, and with the other deſcribe the Arch croſſing the Arch O P in N, from which Interſection a Line drawn by a Rule to A, and another to D, finiſh the Geometrical Square or Quadrant A C D N. *Fig.* 7

IX. *To make a long Square, the length and bredth being given.*

This is ſo like the former, that a particular Figure is not neceſſary to conceive of it. Suppoſe each ſide of the Square in the laſt Problem to conſiſt of 8 ſmall equal parts, and you were to make a long Square whoſe length muſt be equal to a ſide thereof, *viz.* 8. and the breadth half ſo much given you in a Line thus ——4; then when you had drawn the Line C D for the length, and raiſed the Perpendicular at C, you

Fig. 7 you must take the shorter Line given for the breadth, and set it upon the Perpendicular from C upwards to a Point which for distinction we shall call the Point E, imagining it so marked: With the same extent of the Compasses describe an Arch, placing (to that purpose) one Foot in D. Lastly, extending your Compasses to the length of the Line C D, set one Foot in E, and with the other cross the Arch aforesaid. Then a right Line drawn from that Intersection to E, and another from the same to D, complete the long Square.

X. *To make a Rhombus, the Sides being given.*

Fig. 8 If the Angles be not limitted draw any oblique Angle at pleasure, either Acute or Obtuse, as here the Angle B A C, which is Acute. Then let the Line O P be the length of a Side, which being taken with your Compasses, set it from the angular Point A in both Lines to D and E, in which two Points place a foot of your Compasses successively without altering them, *viz.* in D to describe the Arch F G, and in E to describe the Arch H I, crossing one another in the Point K, from which, right Lines drawn to D and E, finish the Rhombus D A E K.

☞ *Note*, if any Angle be given, together with the Side, to limit the shape and content, begin with that, and proceed as before: For you must know, that to make a Rhombus (or Rhomboides) like to another for Figure, or equal to it in Content, it is not sufficient to have the same Sides; for the more oblique the Angles

are,

are, the farther will the Rhombus differ from a Square (and the Rhomboides from a long square) and the leſs will be the Content. But you muſt have an Angle given, (which will produce all the reſt) or elſe a Diagonal Line, which is a right Line paſſing through the Rhombus (or Rhomboides), from one oppoſite Angle to another, and dividing the Figure into two equal Triangles. If the former (*viz.* an Angle) be given, I have ſhewed what uſe is to be made of it. If the latter, (*i.e.* a Diagonal) together with the length of the Sides, you may by taking the length of the Sides with your Compaſſes, and ſetting a Foot in the ends of the Diagonal Line, make a Triangle on the one ſide of the Diagonal, by *Probl.* 6. and then another on the other ſide by the ſame Problem, the Diagonal being a common Baſe to them both; and this will give the Figure exactly.

XI. *To make a Rhomboides, the Sides being given.*

If neither Angle nor Diagonal be given, (for if either of them be limitted, the caſe is ſpoken to in the laſt Problem) make any Angle at adventures as here A B C. Then ſuppoſing the Lines given to be O P and Q R, ſet the length of the longer upon the Line B C from B to D, and the ſhorter on the Line B A to E. Then with the Compaſſes extended from B to E ſet one Foot in D and deſcribe the Arch F G. Likewiſe, with the Compaſſes extended to the length of the Line O P, ſetting one Foot in E with the other deſcribe the Arch H I, interſecting *Fig.* 9

Fig. 9 the former Arch at K, from which Interſection Lines drawn to D and E finiſh the Rhomboides.

XII. *To make a Trapezium, the Diagonal and Lines in order being given.*

*Fi.*10 Let the Line H L be the Diagonal of a Trapezium, whoſe Sides are the Lines A, B, C, D; the Side A being counted the firſt, as that which takes its beginning from the Point H, and the reſt in the order as they are marked Alphabetically.

Then with your Compaſſes ſet to the length of the Line A, place one Foot in H, and with the other deſcribe the Arch E F. Next taking the length of the Line B, with the one Foot of your Compaſſes placed in L, with the other make the Arch G I interſecting the former at K, from which Point of Interſection, Lines drawn to H and L make the Triangle H K L.

Then with the extent of the Line C, ſet one of the Feet of your Compaſſes at L, and deſcribe the Arch O P. Laſtly, ſetting them to the length of the Line D, and placing one Foot of your Compaſſes in H, with the other make the Arch S R interſecting the former at Q; ſo ſhall Lines drawn from Q to L and H make up the Triangle L Q H, and finiſh the Trapezium H K L Q.

I could have been much briefer in this Problem by referring to the ſixth; but this being of very great and frequent uſe, I deſired to be very plain.

XIII. *To*

XIII. *To make a regular Polygon, otherwiſe called a regular multangular, or multilateral Figure, conſiſting of many equal Sides and Angles, viz. above four apiece.*

Being ſatisfied what ſhall be the diſtance between the Center and every Angle, with that diſtance deſcribe a Circle, which being equally divided into as many Parts, as the Figure muſt have Angles (or Sides, for they are equal in number) and Lines drawn from the Points of Diviſion within the Circle from Point to Point, (ordinarily called Chords) the Polygon is finiſhed as in this Diagram.

Suppoſe an Heptagon, or multangular Figure of ſeven Sides, and as many Angles, be to be deſcribed, every Angle being deſigned to be diſtant from the Center A, ſeven Eighths, or three quarters and an half of an Inch; with that diſtance deſcribe the Circle B C D E F G H, which being divided into ſeven equal parts, and Lines drawn from Point to Point, the Heptagon B C D E F G H will be therein included. *Fi.* 11

I ſhall rather leave my unlearned Reader to find out the Points of Diviſion by many tryals, than to puzzle him with the Geometrical way for finding out Chords to that purpoſe; nor ſhall I buſie my ſelf to tell him at large how he may divide 360 by the number of his Angles or Sides, and then finding in his Quotient the Degrees and Parts belonging to every Diviſion, ſet them readily out by a Protractor, or (for want thereof)

by a Line of Chords; for I ſuppoſe him yet ignorant of ſuch things.

I ſhall therefore only tell him thus much: A Line drawn through the Circle at the Center divides it into two equal parts, which being croſſed in the Center by another Line, the Circle will be parted into four equal Parts or Quadrants, and thoſe by halving them into eight Parts. The extent of the Compaſſes whereby the Circle is drawn (uſually called the Radius or Semidiameter) will divide it into ſix equal Parts; two whereof muſt be a third part, and half of one a twelfth part; and theſe ſtill eaſily capable of farther Diviſion.

XIV. *Having the Sides of the Triangles whereof it conſiſteth, orderly given, to make an irregular multangular, or multilateral Figure.*

This will be more fully handled hereafter, when I come to ſhew the method of drawing Plots of Ground: In the interim I will give you a Specimen of an irregular Pentagon.

Fi. 12 Having the Lines of three Triangles given, (which by a Rule hereafter to be mentioned are neceſſary to make up a five-ſided Figure) lay down the greateſt of the firſt, *viz.* 20, from A to B for a Baſe, and by *Probl.* 6. make a Triangle of it and the other Lines 16 and 10, *viz.* the Triangle A B O.

Secondly, you find by the number 20 over the firſt Line of the ſecond Triangle that it is the common Baſe to them both, and therefore by the ſame *Probl.* 6. make the Triangle A B P of the Lines 20, 14, 18. Laſtly,

Laſtly, finding the Baſe of the third Tri- *Fi.*12
angle to be the ſame with 18, one of the Sides of the ſecond, make the Triangle P B Q of the Lines 18, 11, 12: So is the quinquangular Figure finiſhed.

How every Line is to be found in its due order in this or any other ſort of multangular Figures, ſo as to give a true and exact account, not only of the ſuperficial Content, but alſo of the Figure (or ſhape) and ſituation, is to be taught hereafter in the Doctrine and Practice of Protraction.

CHAP. III.

How to find the Superficial Content of any Right-lined Figure, the Lines being given.

AS a Foundation to what I ſhall ſay upon this Subject, there are ſome few Geometrical Principles or Theorems out of *Euclid* and *Ramus*, which I deſire may be remembred; and becauſe underſtanding is a mighty help to memory, I deſign for my Country Reader a kind of ocular Demonſtration, which though not ſo ſtrict and artificial as that which is to be found in the Commentators upon *Euclid* in the quoted places, will be more ſerviceable to him, becauſe more eaſily underſtood.

Theor. 1. *Every Parallellogram being of the ſame length with the Baſe of a Triangle, and of the ſame height with the Perpendicular of that Triangle, is double to it.* Euclid 41. 1.

Fi. 13 Here are two equal Oblongs or long Squares, A B C D and B E F C, and within them two Triangles inſcribed, whoſe Baſes are of the ſame length, and their Perpendiculars (O P and Q R) of the ſame height with the Oblongs. Now each of theſe Triangles being parted into two Right-angled Triangles by their Perpendiculars, then it is plain to the Eye (and from the nature of Diagonals, which ever divide a Parallellogram into equal parts) that the two new Triangles O P D and O P C, which make up the firſt of the given Triangles, are equal to the Triangles D A O and O B C, which make up the remainder of the Parallellogram A B C D. Therefore that Parallellogram is double to that Triangle, which was to be demonſtrated.

In like manner it is evident, that the Parallellogram B E F C is double to the Triangle C Q F, becauſe C R Q is equal to B Q C, and Q R F is equal to Q E F.

Theor. 2. *All Triangles having the ſame Baſe, and lying between the ſame Parallels, are equal.* Euclid 37. 1.

So in our laſt Diagram, the two given Triangles having Baſes of the ſame length, and lying

lying between the ſame Parallels, are evidently equal, becauſe they are demonſtrated to contain each of them the exact half of the Parallellograms, wherein they are inſcribed; and the Parallellograms being equal, their halves muſt be equal alſo.

Theor. 3. *The Sides of a Triangulate (that is, one that hath four or more Sides) are ever two more than the Triangles of which it is made.* Ram. lib. 10. prop. 2.

This is plain by inſpection, if you view again the Figures of the Trapezium and irregular Pentagon, in the 12*th* and 14*th* Problems of the ſecond Chapter. *F.* 10, 12.

Theſe Theorems being allowed to be ſound (as nothing more certain) the Doctrine concerning the Superficial Content of Right-lined Figures might be reduced to a narrow compaſs; for he that knoweth how to husband theſe three Theorems, may eaſily take up theſe *Corollaries, ordine inverſo.*

1. Any quadrangular Figure (regular or irregular) may by a Diagonal be parted into two Triangles; any five-ſided Figure by two Diagonals into three Triangles; and ſix-ſided Figures into four by three Diagonals, *&c.* by *Theor.* 3.

2. 'Tis no matter of what ſhape the Triangle is, as to the rule for meaſuring, for whether it be Right-angled, Acute-angled, or Obtuſe-angled, and whether it have three, two, or no Lines equal; 'tis only the length of the Baſe, and

and height of the Perpendicular, that is conſiderable, by *Theor.* 2.

3. The true meaſure or content of any Triangle, whether alone, or as part of any triangulate Figure of 4, 5, 6, or more Sides (and conſequently of the whole Figure, by ſumming up the content of all the ſeveral Triangles) is found by *multiplying the whole Baſe* of the Triangle by *half the Perpendicular*, or the *whole Perpendicular* by *half the Baſe*; which being a Rule of ſuch infinite uſe in ſurveying, I deſire it may be remembred; and that it may be underſtood, I ſhall give you a plain Example.

Fig. 6 Suppoſe A B the Baſe of the Triangle, belonging to the ſeventh Problem of the foregoing Chapter, to be 44, and the Perpendicular C D to be 20: Whether you multiply 44 the whole Baſe by 10 the half of the Perpendicular, or 20 the whole Perpendicular by 22 the half Baſe, the Product gives the Content 440, as is here apparent.

44	20
10	22
440	40
	40
	440

Having thus given a general method how all Right-lined Figures may be reduced to Triangles, and ſo their Content found out; I might paſs to the next Head concerning Inſtruments, and their uſe; but becauſe there are nearer ways in meaſuring particular kinds of triangulate Figures proper to thoſe kinds, I ſhall briefly touch them.

I. *To find the Content of a Square, or long Square.*

Multiply the length by the bredth, the Product gives the *Area* or Content.

Example of a Square

17 Inches length.
17 Inches bredth.

119
17

289 Square Inches.

Example of an Oblong

25 Feet long.
13 Feet broad.

75
25

325 Square Feet.

II. *To find the* Area *or Content in Measure of a Rhombus.*

Let fall a Perpendicular from one of the obtuse Angles upon the oppoſite Side; that Side multiplied by the Perpendicular, gives the *Area.*

20 Yards the Side.
14 Yards the Perpendicular.

80
20

280 Square Yards.

III. *To*

III. *To find the* Area *of a Rhomboides.*

Divide it into two Triangles by a Diagonal drawn between either pair of the oppoſite Angles, (as ſuppoſe the Acute) then from either of the other Angles (for inſtance, the Obtuſe) let fall a Perpendicular upon that Diagonal; then ſhall that Diagonal, being multiplied by that Perpendicular, give the *Area.*

Example.

19 Rods the Diagonal.
5 Rods the Perpendicular.

95 Square Rods the Content.

IV. *To find the Content of a Trapezium.*

*F.*10 Divide it by a Diagonal into two parts from Angle to Angle, as the Trapezium, *Ch.*2. *pr.*12. is divided by the Diagonal H L; then from the other two Angles (which in that Figure are marked with K and Q) let fall Perpendiculars upon the Diagonal, half the Sum of thoſe Perpendiculars being multiplied by the Diagonal, (or common Baſe) gives the Superficial Content.

Example.

Suppoſe in the Trapezium before mentioned the Diagonal is 33, the Perpendicular from K

13,

13, and that from Q 15, the *Area* or Superfi- *Fi.* 10
cial Content is thus computed:

13 Chains the firſt Perpendicular.
15 The ſecond Perpendicular.

28 The Sum of both Perpendiculars.

14 Their half Sum.
33 The Diagonal.

42
42

462 Square Chains the *Area.*

V. *To find the Content of a regular Polygonial, or multangular Figure, otherwiſe called multilateral.*

Draw a Line from the Center to the middle of any Side; half of the Perimeter (or of all the Sides) being multiplied by that Line beforementioned, gives the Content.

VI. *To find the Content of an irregular Polygon, or many ſided Figure.*

Divide it into Trapezia's and Triangles by Diagonals, then find their Content ſeverally, and Sum up all together; which that you may better apprehend,

Suppoſe the Polygon belonging to the laſt *Fi.* 12
Problem of the ſecond Chapter were given without the Diagonals A B and B P; then by drawing

*Fi.*12 ing thoſe Diagonals, the Figure is divided into three Triangles, whereof two being upon the ſame Baſe AB make up a Trapezium, whoſe Content may be found, juſt in the ſame manner as was taught even now in the fourth Rule, having found the Perpendiculars from O and P falling upon the Line AB. Then there remains the Triangle BQP, whoſe Content may be found by the general Rule concerning Triangles, having found the Perpendicular falling from Q on the Line BP; and then having added the Content of that Triangle to the Content of the Trapezium, you have the *Area* of the whole polygonial Figure.

☞ But now methinks I ſee (as it were) my Country Student ſcratching his Head, and wiſhing for an opportunity to propound two doubts to me.

1. Why I called the Numbers correſpondent to my Lines by divers denominations, as Inches, Feet, Yards, Rods; and ſometimes by none at all but propounding the Numbers abſtractly.

2. How I came to know how many of thoſe Meaſures (whatever they be) are repreſented by the Lines given, and Perpendiculars found.

To the former I anſwer, I am not yet teaching how to meaſure Lines (that work is preſently to follow) but what Lines of Figures are to be meaſured, and the meaſures of thoſe Lines being known (or ſuppoſed) how the Content upon thoſe real (or ſuppoſed) Grounds may be found, and to this purpoſe I might call the Numbers repreſented by the Lines, Inches, Feet, Yards, or any other Meaſures at pleaſure;

pro

provided I called the Squares, to which the *Area* is equal, by the ſame names; for an Inch in length bears the ſame proportion to a ſquare Inch (having length and bredth) that a Mile in length bears to a ſquare Mile. For this reaſon I profeſs not in the Title of this Chapter to teach how to meaſure Figures, (much leſs how to meaſure the Lines of ſuch Figures) but how to find the Content, the Lines being given.

And then to take away the ſecond doubt, know that the Numbers repreſented by the Lines, were either given by thoſe Learned Artiſts from whom I borrowed the Figures, or ſuppoſed by my ſelf as grounds to go upon (as in ſuch caſes is ordinary) or, laſtly, found out to be agreeable thereto by ſome Scale of ſmall equal parts, which he is yet ſuppoſed ignorant of. But now I am going to ſhew him the nature and uſe of two or three plain and cheap Inſtruments, by the help whereof he may with much exactneſs, 1. Meaſure the length of any Lines bounding Right-lined Figures upon the ground. 2. Draw Lines and Figures upon Paper proportionable thereunto (which we call Protracting). 3. Find upon his Paper-Figures the true length of all the deſired Perpendiculars, which ſhall alſo be proportionable to thoſe on the Ground, but much more eaſie to be obtained. And withall, I intend to give him ſuch farther Inſtructions and Cautions for the application of the general and more particular Rules of this Chapter to his peculiar uſe, as will render them (eſpecially ſome of them) ſingularly advantagious.

CHAP. IV.

Concerning Chains, Compaſſes, and Scales.

I. AMongſt the many ſorts of *Chains* uſed for meaſuring Land, three are moſt famous, bearing the Names of their Inventors, Mr. *Rathborne*, Mr. *Gunter*, and Mr. *Wing*, all of them ingeniouſly divided, and uſeful in their kind; but my brevity will give me leave only to deſcribe one, and that ſhall be Mr. *Gunter*'s, being moſt in uſe, and eaſie to be procured.

This Chain contains in length four Statute-Poles or Perches, each Perch containing 16 Feet and a half, or 5 Yards and a half; ſo that the whole Chain is 66 Feet, or 22 Yards long.

This whole Chain is divided into 100 equal parts or Links, whereof 25 are a juſt Pole or Perch; and for ready counting, there is uſually a remarkable diſtinction by ſome Plate or large Ring at the end of every 25 Links, but eſpecially at the preciſe middle of the Chain, which ſhould differ from the reſt in greatneſs and conſpicuouſneſs. Alſo at the end of every tenth Link 'tis uſual to hang a ſmall Curtain-Ring; and if there be at every five Links end a peice of Wire made like the bow of a Link, with a little ſhank an Inch or leſs long, (or ſome ſuch diſtinction) 'tis ſtill better.

When you are to meaſure any Line by this Chain, you need to regard no other Denomination

nation but only Chains and Links, ſet down with a prick of your Pen betwixt them, *e. g.* If you found the ſide of a Cloſe to be 6 Chains and 35 Links long, it is thus to be put down 6 . 35.

But if the Links be under 10, a Cypher muſt be prefixed; ſo 7 Chains 9 Links muſt be thus ſet. 7 . 09.

In the uſing of this (or indeed of any) Chain, care muſt be taken, both to go ſtrait, and to keep a true account; for which purpoſe, it is good that he which goeth before carry in his hand a bundle of Rods, to ſtick down one at the end of the Chain which leads, having firſt ſtretched it well, and that he which follows do not only gather up the Rods to keep the Accompt, but alſo at every remove, mark whether he ſee the Leader directly between his Eye and the Angle, or other Mark he aims to meaſure to; and if need be, call to the Leader to move towards the right or left hand, 'till he ſee him in a direct Line to it.

II. *Compaſſes* are ſo well known, that I need not deſcribe them; only they ſhould be of Braſs, with Steel Points ſmall and neatly wrought, nine or ten Inches long from the Joint to the Points, turning ſo truly upon the Rivet that they may be eaſily opened; and yet ſtand ſo firmly, that an Arch or Circle may be without their ſhrinking deſcribed upon a large *Radius*.

For the form, I would commend above all, thoſe that have large Bows, ſo contrived, that by preſſing them with the hinder part of the Hand, they will gently open, and by the Thumb

and fourth Finger be put together (as others will) ſo that they are manageable by one hand; which is a great convenience for one that at the ſame time ſhould hold his Rule with Scales in the other.

Theſe might alſo be contrived with a Screw to take out one of the Points, to place in the room (upon occaſion) a Black-Lead Pen, or any Pen to draw Circles, with either black or otherwiſe coloured.

And for a Man that would be an Artiſt indeed, it were convenient he were furniſhed with dividing Compaſſes, beam Compaſſes, and triangular ones, for ſeveral uſes not here to be mentioned; but a Country Surveyor may make a good ſhift with ſuch a plain pair as I firſt deſcribed, which Mr. *Wynne*, over againſt the *Rolls* in *Chancery-Lane*, will help him to, with the Chain and Scales, for a ſmall matter.

III. *Scales* are certain Lines divided into equal parts, upon Plates or broad Rules of Braſs or Box, and they are of two ſorts, 1. Plain; 2. Diagonal.

1. *Plain Scales* are made up of two ſmall Lines parallel to one another at a little diſtance, and theſe are divided into great equal parts, which ſignifie Tens, and are noted 10, 20, 30, 40, 50, *&c.* according to the length of the Lines.

They may be of any convenient length, but theſe great diviſions are ſeldom more than Inches, or leſs than third parts apiece.

Again, one of the great Diviſions (or Parts) is ſubdivided into ten equal Parts by ſhort Lines, whereof

whereof that in the middle ſtanding for 5 is longer than the reſt.

According to the Numbers of theſe little parts contained in an Inch, the Scale is named *A Scale of* 10, 11, 12, 16, 20, 24, 30, &c. *in an Inch.* That ſhort one which I give you the Figure of at A is of 10 in an Inch, ſo noted at the top, according as is uſual upon Rules, and Indices of plain Tables. The Line marked O Q, ſeparates Unites and Tens; Unites being taken upward from that Line, and Tens downward; mixt Numbers both ways. *Fi.* 14

As for Example.

7 is the extent of the Compaſſes upon the Scale A from the Line O Q to K; 30 is their extent from the Line ſo marked to O Q; and 27 is their extent from the Line 20, to the ſhort Line K aforeſaid.

Here note, that you muſt not expect to find the Letters O Q or K upon the Scales which you buy, being only marks uſed at pleaſure, to make my meaning plain; and likewiſe that this Scale of 10 in an Inch, and others that are ſmaller (all being compoſed after the ſame manner) are uſually made for more convenient uſe, ſo long as to contain nine or ten (the more the better) of the great Diviſions, ſignifying Tens; though the Figure at A being deſigned for no other uſe than to help your conceptions, extends but a little beyond 30, that length being ſufficient for my purpoſe in this place.

Fi. 14 Theſe plain Scales, eſpecially the ſmaller ſorts of them (ſuch as 24 or 30 in an Inch) are very proper for drawing Figures upon Paper, where the Numbers repreſented by the Lines are not above 100 (for then every Diviſion may be counted as it is upon the Scale) or above, upon a long Scale.

Alſo in the ſurveying of Forreſts, Chaſes, and great Commons, where the Lines are vaſtly long, and the miſtake of a few Links (yea, of half a Pole) is not conſiderable, they may be conveniently uſed, accounting the Tens and Unites to ſignifie ſo many whole Chains, and ſo eſtimating the parts of a Chain with the Compaſſes upon the ſmall Diviſions, which a ſagacious Man may do very near upon one of the larger Scales: But it were much better, in my opinion, for ordinary meaſuring, if the grand Diviſions on the Scale were two Inches a piece, (as I have one upon the Index of my plain Table) for then the ſmaller Diviſions being of five in an Inch, would be ſo large as to be ſubdivided into five a piece, which repreſents 20 Links; and then the half of one of thoſe ſmaller Diviſions ſignifying 10 Links, and the quarter 5, a very ordinary judgment may come very near to the truth by eſtimation.

2. But the *Diagonal Scale* is ſo well known to every Mathematical Inſtrument-maker, ſo eaſie to be procured, and every ways ſo fitted to *Gunter*'s Chain, and our Country-man's uſe, that I cannot but highly commend it.

Of theſe Diagonal Scales, there are two ſorts, the Old, and the New.

By

By the Old one, I mean ſuch as is to be found in Mr. *Leybourn*'s Book, whereof I ſhall preſent you with a fragment, with ſuch a deſcription as may enable you to underſtand the whole. *Fi.*14

1. It is made (as appears by the Figure B) upon eleven parallel Lines equidiſtant, ſo as to include ten equal ſpaces, which are all cut at right Angles by Tranſverſe Lines dividing them all into four equal parts.

2. One of theſe Tranſverſe Lines (*viz.* P R) where it toucheth the firſt and laſt Lines, ſeparates between the Hundreds (or whole Chains) and the Tens, (repreſenting 10 Links apiece) the Chains being numbred *downwards* on the left hand from P only to 3, but on the Inſtrument it ſelf they may go on to 9 or 10, (the Rule being a Foot long) but the Tens (or Decads) *upward* from P to 10.

3. From the Points of Diviſion into Tens upon the firſt Line beginning at P, to the like Points beginning at R in the laſt Line, are nine Diagonal Lines drawn, the firſt beginning at P, and ending at the firſt Diviſion above R. The ſecond beginning at the firſt Diviſion above P, and ending at the ſecond above R. In a word, they are all drawn from one Diviſion leſs from P, to one more from R; by which it comes to paſs, that every Diagonal, by that time it hath paſſed from the firſt Line to the eleventh is a whole tenth part of an Inch (which anſwers to ten Links of the Chain) farther diſtant from the Line P R, than at the Point upon the firſt Line whence it was drawn.

4. Every

*Fi.*14 4. Every one of theſe Diagonals is divided into ten equal parts by the long parallel Lines running through the whole Scale, and numbred on the top from 1 to 9. Whereby it is evident, that the Interſection of any of the nine parallel Lines that are numbred at the head with any Diagonal, muſt be farther diſtant from the Line P R, than the Interſection of the Line next before it with the ſame Diagonal by $\frac{1}{10}$ of $\frac{1}{10}$, that is, by $\frac{1}{100}$, which anſwereth to a ſingle Link of your Chain.

From what hath been ſaid, and inſpection of the Figure B, theſe things plainly follow, which as ſo many clear inſtances will help you to underſtand it fully.

1. The diſtance from P R to the ſecond Diviſion below it anſwereth to two Chains.

2. The diſtance from P R to the eighth Diviſion upward being taken (with Compaſſes) upon the firſt Line of the eleven from P to 8, anſwereth to 80 Links.

3. Conſequently the extent of the Compaſſes from the ſecond grand Diviſion below P to the eighth of the leſs Diviſions upward, is proportionable to 2 Chains 80 Links.

4. The diſtance from P R to the firſt Diagonal being taken upon the parallel Line noted with 9 above anſwereth to 9 Links: Where note, that the firſt Diagonal is not that which is noted with 1, but that which is drawn from the Point P.

5. The diſtance upon the ſame Line from P R to the Diagonal that is marked with 7, is anſwerable to 79 Links.

6. The

6. The extent of the Compaſſes from the bottom of the Figure B upon the ſame Line to the ſame Diagonal, anſwereth to 3 Chains 79 Links. *Fi.* 14

Briefly whole Chains may (by Analogy) be meaſured upon any Line from P R to the grand Diviſion noted with the given Number, Decads alone, or Chains and Decads upon the firſt Line of the eleven where the Diagonals begin. Links alone, Decads with Links, and Chains and Decads with Links, always upon that Line upon which the number of odd Links ſtands at the head of the Scale.

And know, that theſe Directions (*mutatis mutandis*) will as well fit, if half an Inch be only allowed for a Chain, and conſequently all the Diagonals drawn within that extent, as it is uſual (and very commodious for longer Lines) upon the other end of the ſame Rule, the grand Diviſions for Chains going the contrary way, and noted with Numeral Figures in order. It is good therefore when you furniſh your ſelf with Scales, to have Diagonal Scales of both dimenſions on the fore-ſide of your Rule; and upon the back-ſide many plain Scales of equal parts, with a Line of Chords; all which you may have (by enquiring only for the Scales deſcribed in Mr. *Leybourn*'s Book) of Mr. *Wynne* aforeſaid, as likewiſe all other Mathematical Inſtruments.

Having been ſo large for my plain Countryman's ſake, I ſhall not proceed to the deſcription of the new Diagonal Scale, of which you may have the Figure and Deſcription in Mr. *Wing*'s Book: For though it be an excellent good one, as

*Fi.*14 as I know by experience; (Mr. *Hayes* having (at my deſire) furniſhed my noble Friend Sir *Charles Hoghton* with an artificial one of that ſort, when I had the honour of aſſiſting him in Mathematical Studies) yet becauſe 'tis pretty coſtly (if well made), and that before deſcribed will very well anſwer its end, I ſhall at preſent ſay no more of it.

But my Reader may perhaps object to me, that though I have inſtructed him how he may make a Line of an exact length, to anſwer to any number of Chains and Links (given or found by meaſure) upon the Diagonal Scales: I have not yet ſhewed him how to meaſure a Line (as ſuppoſe a Perpendicular) whoſe length is unknown, upon them.

To give him therefore all ſatisfaction (though what I have writ already, might help him to find this out) let us ſuppoſe, that in ſome Figure made according to the Diagonal Scale B of 100 in an Inch, we meet in meaſuring with an unknown Perpendicular equal to the Line in the Margin. Taking it between the Points of my Compaſſes, I firſt try whether it be even Chains, and finding upon the firſt view that it is not, I make a ſecond trial, whether it will prove to be even Decads, or Tens of Links; to which purpoſe I ſet one Foot at 3 Chains in the bottom of my Scale in the firſt Line where the Diagonals begin, and the other Foot reſts in the ſame Line betwixt 6 and 7; whereby I am aſſured, the odd Links above 3 Chains are more than 60, and leſs than 70. And to find how many above 60, I remove the Compaſſes from parallel to parallel

parallel in order, 'till one Foot in the loweſt Line reſting in the end of a parallel, the other will touch ſome Diagonal at the Interſection with that Line which falls out to be at L and O in the Line marked with 7, ſhewing the whole Line, being meaſured by that Scale, to ſignifie 3 Chains and 67 Links. *Fi.* 14

CHAP. V.

How to caſt up the Content of a Figure, the Lines being given in Chains and Links.

HAving deſcribed theſe plain Inſtruments, and in ſome meaſure ſhewed the uſe of them in ſeverals, it were very proper in the next place to teach their joynt uſe in meaſuring and protracting; but becauſe I would have my young Surveyor, before I take him into a Cloſe, able to perform his whole work together, I intend to ſhew him, 1. How he ought to make his Computations; 2. The Grounds or Principles that will juſtifie him in ſo doing.

For the firſt, take theſe Rules:

1. Put down your length and bredth of Squares and Oblongs, and your Baſe and half Perpendicular of Triangles directly under one another, expreſſed by Chains and Links with a prick betwixt them, as was taught before, *Chap.* 4.

2. If the odd Links were under ten, put a Cypher before the numeral Figure expreſſing them,

them, (as there alſo was ſhewed) and if there be no odd Links, but all even Chains, put two Cyphers after the prick.

3. Multiply length by bredth, and Baſe by the half Perpendicular, according to the Rules for finding the Content of Figures, *Chap.* 3.

4. From their Product cut off 5 Figures (accounting Cyphers for ſuch) reckoned from the right hand backward, with a daſh of your Pen, ſo ſhall thoſe to the left hand ſignifie Acres.

5. If thoſe five cut off were not all Cyphers, multiply them by 4, and cutting off five toward the right hand again, the reſt will be Roods or Quarters.

6. If amongſt theſe five Figures towards the right hand that were cut off at the ſecond Multiplication there be any Figures beſides Cyphers, multiply all the five by 40, and cutting off five again by a daſh of your Pen, thoſe on the left hand ſignifie ſquare Perches, Poles, or Roods.

A few Examples will make all plain.

Queſt. 1. *What is the Content of a Square, whoſe Sides are every one of them* 7 *Chains,* 25 *Links?*

Length 7.25
Bredth 7.25

3625
1450
5075

5|25625

5|25625

5|25625
4
1|02500
40
1|00000

Anſw. 5 Acres, 1 Rood, and 1 Perch, as here appears.

Queſt. 2. *In a long Square, whoſe length is* 14 *Chains, and the bredth* 6 *Chains* 5 *Links, what is contained?*

Length 14.00
Bredth 6.05
7000
84000
8|47000
4
1|88000
40
35|20000

Anſw. 8 Acres, 1 Rood, and 35 Perches, as the Work makes it evident.

Queſt. 3. *In a Triangle, whoſe Baſe is* 3 *Chains, and half the Perpendicular* 98 *Links, what is the Content?*

The Baſe	3.00
Half Perpend.	0.98
	2400
	2700
	29400
	4
	1|17600
	40
	7|04000

Anſw. 0 Acres, 1 Rood, 7 Perches, as here is plain.

There be other ways of Computation by Scales, Tables, *&c.* but that this is ſound and demonſtrative, I come now to ſhew by theſe following Steps.

1. It is evident, that in this way of Multiplication the Product is ſquare Links; for every Chain being 100 Links, it is all one to multiply 7.25 by 7.25, or 725 by 725 without pricks, for the pricks ſignifie ſomething as to Conception, but nothing at all in Operation. The Product therefore of the firſt Example was really 525625 Links.

2. Every Chain being 4 Perches long, it follows, that 5 Chains (or 20 Perches) in length,

length, and 2 Chains (or 8 Perches) in bredth, make an Acre, or 160 ſquare Perches; for 20 being multiplied by 8, gives 160.

3. From hence it plainly followeth farther, that there are exactly 100000 ſquare Links in an Acre; for 5 Chains multiplied by 2, is the ſame with 500 Links by 200, which makes 100000. And he deſerveth not the name of an *Arithmetician* that is ignorant of this old plain Rule, *When the Diviſor conſiſts of* 1 *and Cyphers,* (*as* 10, 100, 1000, 10000, 100000, &c.) *cut off from the right hand ſo many Figures of the Dividend as the Diviſor hath Cyphers, accounting them the Remain*; *ſo ſhall the reſt on the left ſide be the Quotient.* It is plain then that 525625 ſquare Links make 5 Acres, and 25625 ſquare Links over.

Thus I have made it clear to a very ordinary capacity, that as far as concern Acres, the Rules for Computation are good. Now for Roods and Perches, though I might turn off my Reader with that known Rule in Decimal Arithmetick: *Multiplying Decimal Fractions by known Parts, gives thoſe known Parts in Integers, due regard being had to the ſeparation.* I ſhall proceed in my plain way thus: If 25625 ſquare Links, which remain above an Acre, do contain any quarter or quarters of an Acre; then if they be multiplied by 4, and divided by 100000, (that is, five cut off from the Product) they will contain ſo many Acres as now they do quarters (or Roods), for any number of quarters multiplied by 4, muſt needs produce the like number of Unites or Integers, and the Diviſion doth only reduce them into the right denomination.

tion. Now 25625 being multiplied by 4, and five Figures being cut off from the Product, the reſult is 1|02500, that is, an Acre and above; which ſhews it was above a quarter before it was multiplied by 4.

And to find how much, (that is, how many ſquare Perches are contained in this laſt remainder) you muſt conſider this 2500, not as ſquare Links remaining above the Rood or Quarter, but as fourth Parts or Quarters of ſquare Links; or (which is all one) as the true number of ſquare Links multiplied by 4, and conſequently being multiplied by 40, (the fourth part of ſquare Perches in an Acre) it muſt as often contain 100000 ſquare Links, (or an Acre) as the quarter of this number 2500, *viz.* 625, ſignifying ſquare Links, containeth ſquare Perches; and ſo it doth, for 100000 divided by 160 (the number of Perches in an Acre) gives 625 as anſwerable to 1 Perch; and 2500 multiplied by 40, gives 100000, or 1 Acre, the five Cyphers being cut off as here is manifeſt.

2500	160) 100000 (625	
40	960	
1|00000	400	
	320	
	800	
	800	
	0	

Some

Some may perhaps wonder, that in ſo ſmall a Manual I ſpend ſo many words about ſuch ordinary things, as in this and the laſt Chapter; but I am moſt afraid, leſt I ſhall not for all my plainneſs be ſufficiently underſtood by ſuch as I purpoſely write for, in things of ſuch neceſſary and frequent uſe: And I deſigned not to make this Treatiſe ſmall by being obſcurely brief in ſubſtantial things, but by leaving out ſuch Curioſities as I thought my Country Friend might well ſpare.

CHAP. VI.

How to meaſure a Cloſe, or parcel of Land, and to Protract it, and give up the Content.

HItherto we have been like Children learning to ſpell, now let us ſet our Syllables together. I mean, let us make uſe of the Inſtructions beforegoing to meaſure a piece of Land, to Plot it, and to caſt up the Content.

All Cloſes, or parcels of Land, are either ſuch as need not to be plotted for finding out their true meaſure, but the Chain alone doth the Work; or ſuch as cannot be conveniently meaſured without Plotting or Protraction.

Of the firſt ſort are the Square and Long Square, known before-hand to be ſuch, or found ſo to be by ſuch Inſtruments as I have not yet

deſcribed, or by meaſuring all the Sides and Diagonals. Theſe Squares and long Squares (I ſay) need no protracting, for you need only to multiply the Chains and Links of the length, by the Chains and Links of the bredth, and ſo proceed as in the firſt and ſecond Examples of the fifth Chapter: But all others, whether Triangles or Triangulate, are to be protracted. I ſhall give Examples therefore in the three ſorts of Figures, triangular, quadrangular, and mult-angular.

But before I proceed to particular Inſtances, let me adviſe the young Practitioner thus: Remember,

1. To begin at ſome notable Angle of the Field, where there is ſome Houſe, Gate, Stile, Well, or the like; or if there be none, then to dig up a Clod, drive down a Stake; or at leaſt, to obſerve what quarter of the Heavens it pointeth towards, whether Eaſt, Weſt, North, or South, and on your Paper mark it with the Letter A, or any other.

2. To go parallel to the ſide of the Field, if Pits, Buſhes, or the like, hinder not, (and if they do, to allow for it) accuſtoming your ſelf to go either *cum Sole*, that is, with your left hand towards the Hedges, Walls, or Pales; or *contra Solem*, with your right hand towards them; and when you go contrary to your uſual cuſtom, note it on your Paper by ſome mark known to your ſelf.

3. To ſet down the Chains and Links of every ſide as you meaſure them, and not to truſt your memory. A Black-Lead Pen will be very proper for this purpoſe.

4. To

4. To take heed (if you have more Scales than one upon your Rule) leſt you confound your ſelf by taking Lines off of ſeveral Scales, or meaſuring Perpendiculars upon wrong ones; for every Line of the ſame Figure muſt be made by the ſame Scale, and the Perpendiculars meaſured by it.

5. To make uſe of a Scale of larger Diviſions when you meaſure ſmall Cloſes, and of ſmaller when you meaſure great ones.

6. To make your Lines and Points where Angles meet, ſmall, pure, and neat.

7. To ſet on your Chains and Links at twice, when any Line is too long for your Scale.

Theſe things being premiſed, I proceed thus:

I. Suppoſe I meaſure a triangular Field with my Chain, beginning at the Eaſtern Angle A, and find the Sides in their order and meaſures to be ſeverally, thus: (I going *cum Sole*) 2.29, 3.45, 4.07.

Making uſe of the leſs Diagonal Scale, (be- *Fi.*15
cauſe the other would make the Figure too large, otherwiſe it were more proper for ſo ſmall a Cloſe) I firſt with my Compaſſes take off the Scale 4 Chains and 7 Links, and ſetting them from A to C draw that Line for the Baſe, becauſe the longeſt of the three: Then I take 2 Chains 29 Links off the ſame Scale, and ſet them in the Eaſtern Point A where I began, and turning the looſe Foot of the Compaſſes above the Line AC, becauſe I went *cum Sole*, I deſcribe (at that diſtance 2.29) the Arch EE.

Next

Fi. 15 Next taking with my Compasses upon the same Scale the extent of 3 Chains 45 Links, I place one Foot in the Point C, and with the other make the Arch F F intersecting the former in the Point B; and drawing the Lines A B and B C, (as was taught in *Ch.* 2. *Probl.* 6.) the Triangle A B C is the Plot of the triangular Field measured.

But before I can give the Content, I must find the length of the Perpendicular, which is done by setting one Foot of the Compasses in B, and extending the other to the Base A C, so as it touch it and pass not over it, (according to *Ch.*2. *Probl.* 7.) for then the length of the Perpendicular is between the Points of the Compasses, and being applied to the same Scale by which the Triangle A B C was made, it appears to be 1 Chain 42 Links. With the half whereof I multiply 4.07 the length of the Base, and proceeding in my Work as was shewed in the last Chapter, the Content appears to be 0 Acres, 1 Rood, 6 Perches, as it is here evident.

The Base	4.07
Half Perpend.	0.71
	407
	2849
	28897
	4
	1|15588

1|15588

1|15588
40
6|23520

II. Suppoſe I were to meaſure a quadrangular or four-corner'd Field, I begin as before at ſome remarkable Angle; and going round the Cloſe *cum Sole*, I find the Sides to be 9.04, 6.72, 8.46, 7.28, and the Diagonal from that remarkable Angle to the oppoſite Angle to be 10.02, I begin therefore to Protract it thus:

Having by the help of my Scale and Compaſſes drawn my Diagonal 10.02 from my remarkable Angle A to C the oppoſite Angle, I make a Triangle of it, and the firſt and ſecond Sides 9.04 and 6.72, according to *Ch.* 2. *Probl.* 6. and another after the ſame method of that Diagonal, and the third and fourth Sides 8.46 and 7.28, ſo have I the Trapezium A B C D. *Fi.*16

Then by the help of my Scale and Compaſſes, I find the Perpendicular of the Triangle A B C to be 6.02, and of the other, *viz.* C D A 6.01, which added are 12.03, whereof the half Sum is 6.01; by which multiplying the Baſe 10.02, and proceeding as formerly hath been ſhewn, I find the Content of the Field to be 6 Acres, 0 Roods, 3 Perches, as is here apparent.

The

*Fi.*16

The Bas	10.02
Half Perpend.	6.01
	1002
	60120
	6|02202
	4
	|08808
	40
	3|52320

Before I pass any farther, let me tell you,

1. Any quadrangular Close, or parcel of Ground whatsoever, having right Lines, may be thus measured, protracted, and computed.

2. The odd measure above Perches is not valuable here, nor in the former Computations, being always under a square Perch; but in multangulars where there be many Remainders, they must be summed up, and the Perches contained in them added to the Content before found.

3. This last, and the following Figures (where I use any Scale at all) are made that they might not be too large, by a Scale of 400 in an Inch, *i. e.* by the less Diagonal Scale, each Chain and Link being counted two.

*Fi.*17 III. If this multangular Figure be conceived to represent a Close of seven Sides, which is to be measured, I begin at the remarkable Angle A, and going round the Close (with the Sun) I find the ides to be in measure 3.11, 2.49, 2.47, 1.77, 4.11, 2.29, 4 37.

Then

Then I measure the four Diagonals, B D, D F, F B, and B G, in the order that I named them, and I find them to be 3.45, 4.97, 4.13, and 4.36, which is as short a way as can be taken, to prevent unnecessary Walks. *Fig.* 17

But when I come to Protract by the help of my Scale and Compasses, I first make the Triangle B C D of the first Diagonal, and the second and third Sides. Then the Triangle D E F upon the second Diagonal, and fourth and fifth Sides, and upon the same Diagonal as a common Base, the Triangle B D F of the first, second, and third Diagonals.

Next of the same third Diagonal, together with the fourth and sixth Sides I make the Triangle B F G, and upon the fourth Diagonal as upon a common Base with the first and last Sides the Triangle A B G, so is the whole Close plotted.

And now it stands visibly reduced into two Trapezia's A B F G and B D E F, together with the Triangle B C D, which I shall not now cast up, having so often shewed how such work is to be done.

But I must acknowledge that this sort of plotting of parcels of Land that have many Angles, requires not only more care and pains, but better skill and memory than to draw Diagonals upon Paper, when the Plot is already taken by the plain Table, or other standing Instrument. I shall therefore to help my young Practitioner in this case, advertize him of two easie ways to help himself, so as to be out of danger of mistakes.

One

*Fi.*17 One way is to divide the Multangular Field into two or more parts as the last might have been by the Diagonal B F; then might each part have been measured severally, as if they had been separated by a Pale, or were sundry Men's Lands parted by a Boundary.

*Fi.*18 Another way that much helps both the understanding and memory, is to draw a rude Draught of the Figure of the Land you intend to measure, not only as to the Sides, but also necessary Diagonals. Then measuring the Lines upon the Ground correspondent to those on the Paper, (which by the help of the Draught may be easily hit) set the Lines as you measure them upon the Lines of the Draught, as if it were the true ones, and when you have finished your measuring, protract it truly. Such as you see here (but it's better larger) will do your business, for 'tis not a Pin matter how rude or false the Lines or Angles be, resemblance being all that is desired.

CHAP. VII.

Concerning the measuring of Circles, and their Parts.

I Have hitherto abstained of purpose from medling with the Circle and its Parts, that I might lay those things close together without unnecessary mixtures, that are of the greatest use.

'Tis

'Tis wonderful rare, if a Land-meeter ever have occasion to measure any Field or parcel of Land, that will prove either Circle, Semicircle, Quadrant, or Sector. Sometimes indeed there will be a little crook in an old Hedge bowing like an Arch: But I have never seen any offer to measure it as a Segment, but always take it as an Angle or Angles.

Yet because it may be expected I should say somewhat of those things, I shall briefly do it.

I. To measure a Circle in the more exact way is to square the Diameter, and to multiply that Square by .7854, so shall the Content be in Integers and Decimals.

But the more usual and quick way (and near enough for any use we shall make of it) is to multiply the half of the Periphery or Circumference by the Semidiameter.

In like manner to find the Content of a Semicircle, Quadrant, or Sector, made up of Semidiameters, and arched Lines, multiplying the half Arch by the Semidiameter, gives the Content.

But that which falls out most frequently in Mensuration (though seldom much regarded save where a curious exactness is required) is that particular sort of Segment, which we call a *Section*, less than a Semicircle, such as this Figure A B C. And to find the Content of it, the center of the Circle whereof this is a Section must be first found out, as here at O, from which Lines drawn to A and B, make up the Sector A O B C; which being measured according to the last Rule, and from the Content *Fig.* 19

*Fi.*19 thereof, the Content of the Triangle A O B ſubtracted, the difference or reſidue is the Content of the Section A B C.

But two Queſtions may be here demanded:

1. How may the Center be found? 2. How may ſuch a portion of Land be truly protracted and computed?

To the firſt I anſwer, that the moſt exact and artificial way is by making a Mark any where in the Arch.

As for Example:

At the Point C; and then (by a Problem known, not only to every Surveyor, but to ordinary Carpenters and Joyners, for finding the center of a Circle, whoſe Circumference will paſs through 3 given points that are not in a right line as ACB) to find the Center O. But if you know not how to do it ſo, croſs the Line A B in the middle, as here it is done by the Perpendicular O C, ſo you may by a few trials find both the due extent of your Compaſſes, and the Point in the Perpendicular that will fit your purpoſe near enough; for if a little errour be committed in making up the Sector, the moſt of it goes off again in the ſubtraction of the Triangle.

II. For the latter you may take this ready courſe: Meaſure the length of both your Lines, (the Chord and the Arch) and their diſtance at the middle of them both. Then when you come to protract, firſt take the length of your right Line from the Scale, and having laid it down, croſs it in the middle at right Angles with a dry Line as in the laſt Figure, ſo ſhall it inter-

interſect the Line A B in the Point E; then *Fi.* 19
from the ſame Scale take the meaſured diſtance between the two Lines in the middle, and ſet it upon that dry Line from the Interſection at E to the Point C. Then by trials find a due place in the dry Line O E C, and ſuch a diſtance with your Compaſſes, that the one Foot reſting in that Line, the other may deſcribe the Arch A C B, and the Section is protracted.

Theſe few hints are as much as I thought neceſſary for my Country Practitioner concerning circular Lines; but if he think otherwiſe, there are large Treatiſes enough, and particularly thoſe I mentioned, whoſe Rules (though ingenious, ſound, and fit to be known by every one that intends to plunge deep into Mathematical Studies) will not (I think) be of that uſe to him in ordinary meaſuring, that I ſhould tranſcend the intended bounds of brevity to tranſcribe them.

CHAP. VIII.

Concerning cuſtomary Meaſure, and how it may be reduced to Statute-Meaſure, & è contra, either by the Rule of Three, or a more compendious way by multiplication only.

WHereas the Statute-Perch or Pole is 16 Feet and a half, and no more, there be Poles of larger meaſure uſed in many places, as of 18, 20, 21, 24, and 28 Feet, yea in ſome 22 Feet and a half. It were therefore very convenient, that our young Surveyor were furniſhed with a Chain fitted to the cuſtomary meaſure of the Country where he lives, as I uſe to make Chains for my ſelf and Scholars of 21 or 24 Feet to the Pole for *Lancaſhire* and *Cheſhire*, where thoſe Meaſures moſt obtain. But becauſe theſe are too large and cumberſome for ſmall Cloſes, it is very convenient, inſtead of one Chain of 100 Links, to make two of 2 Poles apiece, each Pole divided into 25 Links as that of 100 is, which two half Chains may in meaſuring large Fields be tied together by the Loops with Packthread, or joyned by a buttoning Key-Ring for more ſpeedy diſpatch; but in ſmaller we may uſe the half Chain of 50 Links, only taking care that we count not half Chains for whole ones.

And

And in theſe caſes where the Poles are large, and the Cloſes ſmall, it were ſtill more convenient if you had a Chain of 2 Poles only, divided into 100 Links: Only you muſt then take notice, that whereas working by whole Chains and Links, the firſt Multiplication, after five cut off, gives the Content in Acres and Parts. The like work by half Chains and half Links, will give the Content in Roods or quarters of Acres, and parts of ſuch Roods.

But though it is no hard matter (for one that can find out the length of a Link by dividing the number of Feet in a Chain by 100, and provide himſelf of good Iron-wyre, and curtain Rings to make it of, and a ſharp edged File, and round noſed Plyars to make it with) to be furniſhed with ſuch a Chain; yet becauſe every one cannot do this, I ſhall ſhew you how you may eaſily and yet very truly reduce Statute-meaſure into cuſtomary, that ſo the Chain before deſcribed may do your buſineſs all *England* over.

Know therefore (for a ground to go upon) that Acres bear proprotion to one another, as the ſquares of their Poles; and therefore if you multiply 33 the number of half Feet in the Statute-Pole by it ſelf, which gives 1089, and alſo multiply the number of half-feet contained in a Pole of that meaſure you would reduce into; in the ſame manner you may by the Rule of Three reverſe obtain your deſire, making to that purpoſe 1089 the firſt number, the Statute-meaſure the ſecond, and the ſquared half-feet of the Pole given the third. As for example:

Suppose of a Close measured by the Statute-Pole, the length, breadth, and their product be as here represented in the Margin. And it is desired that the Content may be cast up according to our large *Cheshire* measure of eight yards or 24 feet to the Pole or Rood (as we call it:) Then before I cut off any Figures, I consider that in the Statute-pole are 33 half-feet, and in the *Cheshire*-pole 48, then multiplying 33 by 33, and 48 by 48, I have these two square numbers, 1089 and 2304, which together with the said product may be thus placed: 1089 . 672693 :: 2304, and so multiplying 672693 by 1089, and dividing their product, being 732562677 by 2304, the quotient is 317952, from which if 5 figures toward the right hand be cut off, and dealt withal as was taught in the fifth Chapter, the Content by our customary measure of 24 feet to the Pole, will be 3 acres, 0 roods, 28 perches as here appears.

9.33	L.
7.21	B.
933	
1866	
6531	
672693	prod.

3|17952
4

1|71808
40

28|72320

But

But if the lines on the land had been meaſured according to our cuſtom here, of 24 feet to the pole, and the Content muſt have been found according to Statute meaſure: then I muſt have multiplied the product by 2304, and have divided that latter product by 1089. And in the ſame method you may proceed in all or any of the reſt. But the truth is, that though this way be very exact, plain and comprehenſive, ſuting all the cuſtomary meaſures before-mentioned without fractions, which for my Learners ſake I ſtudiouſly avoid, and for that reaſon reduced my poles to half-feet: It is ſomething tedious except he knows how to relieve himſelf by a large Table of Logarithms, or at leaſt a ſet of *Nepair*'s-bones, which I cannot ſtand here to treat of: Therefore to contract the work a little, take notice, that all the cuſtomary poles before mentioned, (ſaving onely thoſe of 20 and 28 feet, which I ſuppoſe are ſomewhat rarely uſed, becauſe I never heard nor read of them (to my remembrance) ſave only in Mr. *Holwell*; all the reſt I ſay, are capable of being divided into half-yards: And therefore if inſtead of ſquaring the half-feet you ſquare the half-yards of both poles, and work with them, you will attain the ſame end without any regardable difference, the ſmall diverſity that there is being uſually in the uſeleſs remainders, not at all affecting the deſired Quotient that gives the anſwer near enough for uſe.

As for Example.

11
11
11
11
121
16
16
96
16
256

If I had ſquared 11, the number of half-yards in the Statute-pole, which would make 121, and alſo 16 the number of half-yards, in our *Cheſhire* Pole, which would make 256, as appears in the Margin, and then multiplied the firſt product 672693 by 121, the ſecond product would have been 81395853, which being divided by 256, the Quotient would have been (as before) 317952. And this way is in a manner coincident with Mr. *Holwells* firſt Method.

Take notice alſo further once for all, that whether you uſe either of theſe or the following Methods, you need not reduce the particular Squares, Triangles, or Trapezia's ſeverally; but ſum up all their products together, and then reduce all at once.

But if you would reduce Statute-meaſure into Cuſtomary by Multiplication onely, take notice of this preſent Table following.

The Content by the Statute-pole being multiplied by	·84027 ·68062 ·61734 ·53777 ·47265 ·34725	Gives the Content by the Pole of	18 20 21½ 22 24 28	Feet.

The

The Uſe of this Table.

When you have multiplied Lengths by Breadths, or Baſes by half Perpendiculars, multiply theſe Products by the Decimal Fractions anſwering to the Cuſtomary-meaſure into which you would reduce Statute-meaſure, and from that latter product, firſt cut off five places towards the right-hand as not to be regarded, (being only parts of a ſquare Link;) then cutting off 5 more, and proceeding to multiply by 4, and then by 40, as hath been often ſhewed, you will have the Content by that Cuſtomary-meaſure.

Example.

Suppoſe once more the length of a Cloſe meaſured by *Gunter*'s Chain, and multiplied by the breadth meaſured alſo by the ſame, produced 672693 ſquare Links; and it is deſired that the Content may be given in *Cheſhire* meaſure of 24 to the Pole: You muſt multiply 672693 by .47265, the decimal fraction anſwering to 24 feet, and from that product being 31794834645 cut off and caſt away 5 places, and the reſt being 317948, are in the uſual way eaſily reducible into 3 Acres, 0 Roods, 28 Perches, as here appears, agreable to what it amounted to in the former Method.

3|17948
4
|71792
40
28|71680

But

But if you measured by a Chain of Customary Poles, and desire to know what the Content is in Statute-measure; this following Table is for your purpose.

	Feet.			
The Content measured by the Pole of	18	being multiplied by	1.19008	gives the Content by the Statute-pole.
	20		1.46923	
	21		1.61983	
	22½		1.85950	
	24		2.11570	
	26		2.87970	

To understand which, take this Example.

Suppose the length and breadth of a long Square being measured by a Chain of 24 feet to the Pole, and multiplied together, make their product 317952, let this be multiplied by 2.11570, which answereth to 24 feet, and the latter product will be 6726910464o, from which if you cut off and cast away 5 places towards the right-hand, the remainder is 672691, which in the usual way is easily reduced to 6 Acres, 2 Roods, and 36 Perches, as you see here.

6	72691
	4
2	90764
	40
36	30560

One thing more and I have done with this business of Reduction: If the Content to be reduced, be given cast up into Acres, Roods, and Perches, reduce all into Perches, and then in other respects work as before either by the Rule of Three, or by this last Method of Multiplication

plication only. So ſhall you have the Content in ſquare Perches according to the Meaſure deſired, which you may reduce into Acres by dividing them by 160, and if any thing remain, that remainder being divided by 40, will give you the Roods in the Quotient, and the latter remainder the number of ſquare Perches.

For tryal of which Rules, mind the Anſwer to theſe two following Queſtions wrought all three ways.

Queſt. 1. *How many Acres, Roods and Perches, according to the Pole of* 18 *Feet, are contained in* 5 *Acres* 3 *Roods, and* 11 *Perches, Statute-meaſure?*

Anſw. 4 Acres, 3 Roods, and 22 Perches, as here appears.

I. Method.

		A	R	P.
33	36	5	3	11
33	36	4		
99	216	23		
99	108	40		
1089	1296	931		

1089 . 931 :: 1296

1089

8379

7448

9310

1013859

1296)

1296) 1013859 (782
9072
10665
10368
2979
2592
387

160) 782 (4
640
40) 142 (3
120
22

II. Method.

11 12
11 12
11 24
11 12
121 144

121 . 931 :: 144
121
931
1862
931
144) 112651 (782
1008
1185
1152
331
288
43

160)

160) 782 (4
640
40) 142 (3
120
22

Or thus:

40) 782 (19
40
382
360
22

4) 19 (4
16
3

III. Method.

.84027
931
84027
252081
756243
782.29137

160) 782 (4
640
40) 142 (3
120
22

Queſt. 2. *How many Acres, Roods and Perches, of Statute-meaſure are contained in* 8 *Acres,* 3 *Roods (or Quarters) and* 21 *Perches of* 21 *Feet to the Pole?*

Anſw. 14 Acres, 1 Rood, and 21 Perches, as appears by the three following works in the ſeveral Methods.

I. Method.

A	R	P.			
8	3	21	42	33	
4			42	33	
35			84	99	
40			168	99	
1421			1764	1089	

1764 . 1421 :: 1089

1764

5684
8526
9947
1421

1089) 2506644 (2301
2178

3286
3267

1944
1089

855

160) 2301 (14
160
701
640
40) 61 (1
40
21

II. Method.

14 11 196 . 1421 : : 121
14 11 196
56 11 8526
14 11 12789
196 121 1421

121) 278516 (2301
242
365
363
216
121

160) 2301 (14 95
160
701
640
61

40) 61 (1
40
21

III. Method.

1.61983
1421
161983
323966
647932
161983
2301.77843

160) 2301 (14
160
701
640
40) 61 (1
40
21

CHAP.

CHAP. IX.

How a man may become a ready Measurer by Practice in his private Study, without any ones assistance or observation, till he design to practice abroad.

THis Art above all parts of the Mathematicks, is burdened with two Inconveniences to the young Practitioner: The necessity of having one to assist him in measuring his Lines with the Chain, as oft as he would practice his skill, or get more, and the exposing of his unreadiness to the view of meddlesom people, while he is yet raw and unexperienced, as every one must needs be at first: Both which may in good measure be avoided by this easie knack.

Take a small Packthread, and by knots about ½ Inch asunder divide it into an 100 parts, as *Gunter*'s Chain is divided; So shall these small divisions between the knots answer to Links; and if they be not exactly of one length, the matter is not very weighty, but the more equal and short they are, the better. Having this String thus prepared, and marked with longer and shorter pieces of thread tied in the knots, so as you may readily see where is the middle, and where your divisions of 25 Links, and the smaller divisions of 10 links begin and end; make all sorts of Figures in your Study or Chamber, by marking places, or sticking Knives

or Bodkins at pleasure for Angles, accounting the streight lines betwixt them for sides, and so measure the Figures by your knotted string, and cast up the Content by a Scale. This Work you may manage with your own hands in private, and so make your self very quick and ready when you begin to measure for good and all; as I once made a full experiment, and it was thus:

When I first began to instruct Youths in Mathematical Learning in *Warrington*, some of my Boy's Parents desired a sensible demonstration of their Sons proficiency in somewhat that they themselves could in some measure understand; and particularly pitched upon measuring a piece of Land: Whereupon I took four or five of my Scholars to the Heath with me, that had only been exercised within the Walls of the School, and never saw (that I know of) so much as a Chain laid on the ground: and to the admiration of the Spectators, and especially of a skilful Surveyor then living in the Town, they went about their work as regularly, and dispatched it with as much expedition and exactness, as if they had been old Land-Meters.

CHAP.

CHAP. X.

How to meaſure a piece of Land with any Chain of what length ſoever and howſoever divided; yea with a Cord or Cart-Rope, being a good Expedient when Inſtruments are not at hand of a more Artificial make.

IF you can procure a Chain, and find it is not divided as before hath been ſhewed, but into Feet or quarters of Yards, or any ſuch vulgar diviſions, make no reckoning of the diviſions at all, but meaſure it as exactly as you can to find out the true length of the whole Chain, and if it fit none of thoſe lengths mentioned in the 8th Chapter, nor any of their halfs, make it to fit, by taking off a Link or two, or piecing it out with a ſtring; then dividing the length of that Chain by 100. or the half of it by 50, find the true length of a Link according to our artificial diviſion, and having got a long ſtick or rod, ſet as many of thoſe link-lengths upon it as it will hold; Then may you meaſure all the whole Chains by your regulated Chain, and the odd links of every line by your divided ſtick or rod, as is manifeſt in this Example following.

Being far from mine Inſtruments, and requeſted by a friend to meaſure him a Cloſe, I procure a pair of Compaſſes, an ordinary Carpenters Rule of 2 foot, divided into Inches and

quarters, and meeting also with a piece of an old Chain seemingly divided into feet, I measure it by the Rule, and finding it to be 45 feet long, and some odd measure, I piece it out with a pretty strong Cord that will not stretch much, to 48 feet exactly; then will it serve me for half a Chain of 24 feet to the Pole: This 48 I multiply by 12 (the number of Inches in a Foot) and that product being 576, I divide by 50, the number of links in half a decimal Chain, and the Quotient is $11\frac{26}{50}$ Inches, or 11 Inches and an $\frac{1}{2}$, and a trifle over: So then dividing a long stick throughout into such parts, each containing 11 Inches and an half, besides the breadth of the nicks, I am provided of Tools to measure Lines to a Link with exactness enough.

In like manner would I proceed with a Cord or Rope, having fitted them to some known length or other: And then for protraction it were easie with the Compasses to make a plain Scale of a large sort, either upon Paper, or an even piece of Wood; this for once may serve a mans turn well enough.

Besides there is a way of measuring the Perpendiculars of Triangles and Trapezia's upon the ground it self, so as to prevent the necessity of a Scale; for if you have a little Square with an hole in it, to turn upon the head of a little stick, which you may fix where you please, as you are measuring the Base of a Triangle, or the Diagonal of a Trapezium, you may by a very few trials find the place where the one Leg will be just in the Line which you are measuring, and the other point at the Angle from which the Perpendicular falls on it, and then the space between

between your Stick and that Angle truly measured, is the Perpendicular.

If you have not such a Square, a square Trencher, or any end of a Board that hath one right Angle, and two true sided, will supply the want of it.

And now that I am mentioning this way of measuring, I shall make bold to add, that this is a good way, (and as such ordinarily used by that general Scholar and reverend Minister Mr. *Samuel Langley* of *Tamworth* in *Staffordshire*, whose ancient acquaintance I have long esteemed both mine happiness and honour) to measure a Trapezium thus, though it be protracted afterwards; for by measuring the Perpendiculars as aforesaid, and observing at how many Chains and Links end the said Perpendiculars meet the common Base, the whole Trapezium may be truly protracted, without going about it; this little Square competently supplying the place of an Instrument, which is usually called a Cross or Square, made up (as it were) of two small Indices, like those for a Plain-Table (but much less) with fore-sights and back-sights, and cutting one another at right Angles, put together, and having an hole at the Center, like those things which here in *Cheshire* we call *Yarndles*, being used by Country Housewives in winding of their Yarn.

CHAP.

CHAP. XI.

Concerning dividing of Land Artificially and Mechanically.

WEre it ſutable to my Deſign or Humour to be copious or curious, I had here a fair opportunity; for four or five modern Survey-books of the beſt Accompt lying open before me, would tempt me to tranſcribe abundance of ingenious things; but for reaſons often hinted before, I ſhall confine my ſelf to a few plain things that will competently do this buſineſs.

1. To divide a Triangle into any parts required; divide the Baſe as the Demand imports; then ſhall Lines drawn from the Points of Diviſion to the oppoſite Angle finiſh the Diviſion of the Triangle.

Example.

Fig. 20 A C, the Baſe of the Triangle A B C, being divided into 12 equal parts, a Line drawn from the Angular point B to the point 6 divides the Triangle into two equal parts. 2. Lines drawn to 4 and 8 divide it into three equal parts. 3. Lines drawn to the Points noted with 3, 6, and 9, divide it into four equal parts; and ſo Lines drawn to 2, 4 6, 8, 10, divide it into ſix equal parts.

Alſo it is very obvious, that if the ſame Triangle were ſo to be divided, that the one part ſhould be double to the other; a Line drawn from

from B to 4 or 8, doth the work. Or if it be required to divide it into two parts, ſo as the one ſhall be triple to the other, a Line drawn from B to 3 or 9, compleats the Work. So alſo a Line from B to 2 or 10 divides it into two parts, whereof the one is quintuple (or five-fold) to the other, and a Line from B to 1 or 11, divides it into two parts, whereof the one is 11 times as large as the other. *Fig.* 20

Further yet, If it were required this Triangle ſhould be ſo divided, that the two parts ſhould in quantity bear proportion, as 5 and 7, a Line from B to 5 or 7, doth that feat.

But to deal plainly with you, I muſt confeſs that ſometimes the Diviſion will be a little more intricate than thus, yet not ſuch, but that the ſeeming difficulty may be eaſily overcome, by obſerving the method wherein I ſhall ſatisfie the following demand.

Suppoſe a large Triangle of common Land be to be divided amongſt three Tenants A, B, and C, according to the quantity of their Tenements, A having 19 Acres of Land to his Tenement, B 13, and C 7, the Baſe of the Triangle being found by meaſure to be 17 Chains and 27 Links; and the Demand is, where the Points of Diviſion muſt be placed in the Baſe, ſo as Lines drawn from thence to the oppoſite Angle, ſhall truly limit each mans part?

To anſwer this, let us add 13 and 07 to 19, (as in the Margin) and they give 39: So is the work plainly reduced to the Rule of Fellowſhip; and therefore to find every mans diſtinct portion, we need only to multiply the Baſe by his par-

19
13
07
39

*Fi.*20 particular number, and divide that product by 39, the sum of all their numbers, as here is plain:

A

39 . 17.27 :: 19

19

15543
1727

39) 32813 (841 14/39
312

161
156

53
39

14

B

39 . 17.27 :: 13

13

5181
1727

39) 22451 (575 26/39
195

295
273

221
195

26

C

39 . 17.27 :: 7

7

39) 12089 (309 28/39
117

89
78

11

From

From theſe Operations it is plain, that if we ſet off from the Angular point where the Baſe begins, 8 Chains 41 Links, and a little above the third part of a Link upon the Baſe for A, and where that ends, 5 Chains and 75 Links and $\frac{2}{3}$ of a Link for B, and conſequently leave between this ſecond diviſion and the other end of the Baſe 3 Chains and almoſt 10 Links for C; Lines drawn from thoſe points of diviſion to the oppoſite Angle, will give each man his due. *Fi.*20

What I have ſaid touching the diviſion of Triangles upon their Baſes, will with a little variation ſerve for the dividing of all ſorts of Parallellograms, whether Squares, Long-ſquares, Rhombus's, or Rhomboides's: all the difference is, that in ſtead of drawing Lines from Points in the Baſe to the oppoſite Angle, you muſt draw parallel Lines from Points in one oppoſite ſide to another, as will be ſufficiently plain by this one Inſtance.

Suppoſe the ſquare Figure in the 8th Prop. of the ſecond Chapter to repreſent a Cloſe of ſix Acres, and I am to cut off an Acre at the ſide A C, having ſet off the 6th part of the Line C D, from C towards D, and alſo from A towards N, a Parallel drawn between thoſe Points, takes off exactly a 6th part, or an Acre. *Fig.*7

If it be not thought convenient (as in ſome caſes it is not) to cut off a piece ſo long and narrow, you may by the Rule of Three find what other length of any greater breadth will limit an equal quantity to it. Or you may multiply the breadth by 2, 3, or any other, and divide the length by the ſame number that you

Fig.7 multiplied the breadth by. Or laſtly, if you ſet out a double proportion that is $\frac{2}{6}$ or $\frac{1}{3}$, from C towards D, and from the Point where it falleth, draw a Line to the Angle A, you will have a Triangle equal to $\frac{1}{6}$ of the Square A C D N.

But to return to Triangles, (the moſt ſimple and primitive of all Rectilinears, and therefore the moſt conſiderable in this caſe of partition, as giving Laws often to the reſt:) It may fall out, that a Triangle muſt be divided (convenience ſo requiring) by a Line from ſome Point in a ſide, ſo as that Line may either be parallel to ſome other ſide, or not parallel to any. For the former caſe take this Example following out of Mr. *Wing*, *Lib.* 5. *Prob.* 5.

Fi.21 Let A B C be a Triangle given, and it is required to cut off $\frac{1}{5}$ by a Line parallel to A B. Firſt, on the Line A C, deſcribe the Semi-circle, A E C, whoſe Diameter C A divide into five equal parts according to the greater term, and upon three of thoſe parts (the leſſer term) erect the Perpendicular D E, which cutteth the arch Line in E; then ſet the Line C E from C to F, and from thence draw the Line E G parallel to A B; ſo will the Triangle C G F contain $\frac{1}{5}$ of the Triangle A B C, as was required.

Fi.22 Now for the latter caſe, when the line of partition goes not parallel with any ſide, take this Example:

Let A B C be a Triangle given to be divided into two parts which ſhall bear proportion

tion to one another, as 3 and 2, by a Line drawn from the point D in the Baſe, or Line A C. *Fi.*22

From the limited point D, draw a Line to the Angle B; then divide the Baſe A C into five equal parts, and from the third point of Diviſion draw the Line to E, parallel to B D. Laſtly from E draw the Line E D. So ſhall the Trapezium A B E D be in content, as 3 to 2, to the new Triangle D E C.

I have now done with the Diviſion of Triangles, when I have added theſe three Advertiſements.

1. You muſt be ſure to take very exactly the diſtance of every point, where a dividing line cutteth any ſide, to one of the ends of the ſame ſide, as in this laſt Figure, the diſtances B E and A D, which diſtances being applied to the Scale by which the Triangle was protracted, will ſhew at how many Chains and Links end you are to make your dividing Line on the Field it ſelf.

2. The proportions by which you are to divide, are not always ſo formally given as in the former examples, but are ſometimes to be found out by Arithmetical working, as in this caſe.

Suppoſe a Triangular Field of 6 Acres, 2 Roods, and 31 Perches, muſt be divided, ſo as the one of the two parts ſhall be 4 Acres, 3 Roods, and 5 Perches, and the other (conſequently) 1 Acre, 3 Roods, and 26 Perches, reduce both meaſures into Perches, and the one will be 765, and the other 306. Their Sum is 1071, which by their common meaſure being

*Fi.*22 reduced into their loweſt terms of proportion in whole Numbers, will be 5, 2, and 7, which ſhews that the Triangle being divided into 7 equal parts, the one muſt have 5 of thoſe 7 parts, and the other 2. And obſerve, that it will be ſufficient to find the common meaſure between the Sum of the terms and either of the terms; the method whereof is ſhewed in every Arithmetick Book for reducing Fractions into their loweſt terms.

But if my unlearned Reader cannot skill of that work, he may multiply either of the parts; (as ſuppoſe 765) by the length of the Baſe, which we will ſuppoſe to be 8 Chains and 75 Links, or 875 Links: and that product divided by 1071, (the Content of the whole Cloſe in Perches) gives by the Rule of Three direct, 625 Links, or 6 Chains and 1 Pole, the true diſtance from either end of the Baſe, that his mind or occaſions may direct him to begin with, to the point of Diviſion; for the Diviſion muſt be not only for proportion or quantity, but alſo as to poſition or ſituation of parts upon the Paper, as it is required to be on the Ground.

3. In theſe and all other diviſions of Land, where a ſtrict proportion in quantity is to be obſerved, you muſt have reſpect to the Rules hereafter following, concerning meaſuring of uneven Ground, *Chapter* 15. eſpecially if one part prove much more uneven than another; and if there be an uſeful Pond or Well to draw your Line of Diviſion through it; but if it be an unuſeful Pond, Lake or Puddle; or if there be any boggy or barren ground, that muſt be caſt out

out in the diviſions; meaſure that firſt, and ſubtract it from the Content of the whole Cloſe, and then lay the juſt proportion of the remainder on that ſide that is free from it, that the other may have its juſt part alſo, beſides that which is uſeleſs.

What hath been ſaid, with an ordinary meaſure of diſcretion, may ſufficiently inſtruct a young Artiſt to divide Triangles, Parallellograms, and regular Polygonials, in an artificiall way: but becauſe many Cloſes and open Grounds are Trapezia's, and many irregular Polygons, and even thoſe that are regular enough, may fall under an irregular diviſion, in regard of the quality of the Land, Woods upon it, or Quarries in it: Or the conveniencies of ways, Currents of Water, ſituation in reſpect of adjacent Lands, &c. I ſhall propoſe a Method, which though it hath ſomewhat of the Mechanick in it, will be ſingularly uſeful in ſuch caſes.

Let A B C D be a Trapezium to be divi- *Fig.* 23
ded betwixt a young Heir and his Mother, ſo as his part may be double to hers. Having by the Diagonal B C divided it into two Triangles, I find the Content of the Triangle A B C to be 138550 ſquare Links, and the Triangle B C D to contain 103468, in all 242018 ſquare Links, which if reduced, as hath been formerly taught, would amount to 2 Acres, 1 Rood, and 27 Perches: (for which ſee *Chap.* 5.)

But as to my preſent Work, they are in a better Order already. Dividing then 242018 into three parts, each of them is 80676 and $\frac{2}{3}$,

*Fi.*23 two therefore of thoſe third parts muſt contain 161353 and $\frac{1}{3}$, which $\frac{1}{3}$ being inconſiderable, I regard not.

Then reſolving to lay out the double part towards the Line B D, I ſtrike at adventures the Line E F, and meaſuring the Trapezium bounded by that Line, and the oppoſite ſide B D, together with the Interjacent parts of the Lines A B and C D, I find it to contain 119140 ſquare Links, but becauſe it ſhould have been 161353, I ſubtract 119140, out of 161353, and their difference is 42213, and perceiving that the Lines A B and C D are very near parallel, and finding their diſtance where they are cut by the Line E F, to be 326 Links, or 3 Chains and 26 Links, I divide 42213 by 326, and the quotient is 129 Links and almoſt half, at which diſtance I draw the Line G H parallel to E F; ſo ſhall the Trapezium G B D H be the Heirs part.

Another way whereby this may be performed, is thus: Finding the Triangle A B C, to contain 138550 ſquare Links, ſubtract it out of the Heirs part, *viz.* 161353, the difference 22803, ſhews how many ſquare Links muſt be taken out of the Triangle B C D, and added to the Triangle A B C: Which to perform with all neceſſary exactneſs; ſuppoſe the ſide or line B D to be the Baſe, which by meaſure proves to be 344 Links, or 3 Chains and 44 Links. Say by the Rule of Three direct, If the whole Content of the leſſer Triangle, *viz.* 103468, give 344; what ſhall 22803 give? ſo will the reſult be 75 links, and ſomewhat more then $\frac{1}{2}$ of a Link;

a Link; for 22803 multiplied by 344, gives 7844232, which being divided by 103468, the Quotient is 75 $\frac{8131}{103468}$; or (according to decimal Diviſion) 75.8131, which is (as I ſaid before) ſomewhat more than 75 Links and $\frac{4}{5}$, wherefore extending your Compaſſes upon the Scale to almoſt 76 Links, ſet that diſtance upon the Line B D, from B to I, and draw the Line C I: ſo ſhall the Trapezium A B I C be double to the Triangle I C D, within ſo ſmall a matter as is not worth regarding, though the Land were a rich Meadow. Fi. 23

I hope I need not ſtand to tell any man of ſenſe, that if he pleaſe he may begin with the leſs part, and take out that: or if there be many Partners, he may divide betwixt any one, and all the reſt (putting their parts together) and then by the ſame method ſubdivide amongſt them till each hath his due ſhare; nor to ſpend many words in telling him he muſt ſubtract where he hath by a ſeparating Line at adventures, or by chooſing out a Triangle, taken too much: as I added, when I took too little. Nor laſtly, that theſe methods are not only applicable to Trapezia's, but to any triangular Figure whatſoever, whether regular or irregular.

CHAP.

CHAP. XII.

Concerning the Boundaries of Land, where the Lines to be meaſured muſt begin and end.

IF there be no agreement between the Parties concerned, (for if there be, that muſt be obſerved) Reaſon and Cuſtom are the Surveyor's Guide.

The Farmer ſpeaks lowdly, that when a piece of Arable or Meadow-land is let for a year to be ſown or mown, no more ſhould be meaſured nor expected to be paid for, either to the Letter or Workmen, than the Plow or Sythe can go over. So alſo when a parcel of Land is let for Paſture by meaſure to a Farmer, it ſeems very reaſonable, that all and only ſo much ſhould be meaſured as is uſeful to that purpoſe.

But Commons to be encloſed are uſually meaſured (except it be otherwiſe agreed) to the uttermoſt bounds of every mans particular proportion, without any allowance for Ditch or Fence; every man being to make them upon his own of what breadth he pleaſes: Nor is this unreaſonable, for 'tis as good for one as another, and the rate paid to the Lord is uſually very little, ſometimes nothing.

It is alſo very uſual in meaſuring betwixt Lord and Tenant, in caſe of Leaſes for Lives, and

and long terms of years, to extend the Lines to the utmoſt bounds of the Tenants claim, taking in the very Walls, Hedges, and Ditches: but this is accounted very hard, and oft proves very unequal among the Tenants of a Lordſhip; ſome being forced to make much more waſte of their Ground this way, than others that hold as much or more. But where the Cuſtom obtains, the Surveyor muſt obſerve it: For it is others work to appoint what muſt be meaſured, and his only to meaſure truly what is ſo appointed. A good Landlord may (and will be apt) to conſider it in his Rates, and a bad one 'tis like will be tenacious of a Cuſtom to his own advantage.

Laſtly, In caſe of Sale by meaſure at a rate agreed upon *per* Acre (no Boundaries being ſpecified in the Bargain) the Rule (as I had it from an old famous Surveyor many years ago) is to extend the Lines to the quick Wood-row, that is, as Reaſon prompts me to underſtand him, to the place where the quick-wood actually groweth, or where according to cuſtom it ought to be ſet.

I confeſs theſe things are trivial, but yet more neceſſary to be known than many artificial things much ſtood upon, for a young Artiſt, whom ignorance of theſe things may expoſe to conſiderable miſtakes in practice.

CHAP.

CHAP. XIII.

Containing a description of the Plain-Table, the Protractor, and Lines of Chords.

THough what hath been already said, may competently suffice to instruct my young Artist in measuring a Close of Land, yet to advance him a degree higher in useful knowledge, I shall take occasion to describe unto him the Plain-Table, which with Mr. *Wing* I account the best of all fixed Instruments: This Instrument consists of several parts.

1. The *Table* it self, which is a Parallellogram of Wood fourteen Inches and an half long, and eleven Inches broad, or there abouts, and for necessity may be made by an ordinary Country-workman of one Board: but for neatness, convenience of carriage, and freedom from warping, it is usually made of three little Boards joyned together side-ways, with a ledge at each end to hold them fast together, and upon the middle Board a Socket of Brass fixed with three Screws, and with a fourth to be fastned on the head of a three-legged Staff; of which anon.

2. A Frame of Wood fixed to it, so as a sheet of Paper being laid on the Table, the Frame being forced down upon it, squeezeth in all the edges, and makes it lie firm and even, so as a Plot may be very conveniently drawn upon it: this is usually made with Joynts for more easie carriage, but a plain one may suffice.

Upon

Upon one ſide of this Frame ſhould be equal diviſions, for drawing parallel Lines both longwiſe and croſs-wiſe (as occaſion may require) over your Paper, and on the other ſide the 360 degrees of a Circle projected from a Center of Braſs conveniently placed in the Table.

3. A Box with a Needle and Card, to be fixed with two Screws to the Table, very uſeful for placing the Inſtrument in the ſame Poſition upon every remove.

4. A three-legged Staff to ſupport it, the Head being made ſo as to fill the Socket of the Table, yet ſo as the Table may be eaſily turned round upon it, when 'tis not fixed by the Screw.

5. An Index, which is a large Ruler of wood, (or rather braſs) at the leaſt ſixteen Inches long, and two Inches broad, and ſo thick as to make it ſtrong and firm, having a ſloaped edge, (by which we draw the Lines) called uſually the fiducial edge, and two ſights of one height, (whereof the one hath a ſlit above, and a thread below, and the other a thread above and a ſlit below) ſo ſet in the Ruler, as to be perfectly of the ſame diſtance from the fiducial edge. Upon this Index 'tis uſual to have many Scales of equal parts, and there might be a Diagonal Scale if the Inſtrument-maker pleaſe, and Lines of Chords of ſundry lengths: but if you have ſuch a Scale as I before deſcribed, you need not to have them here.

The *Protractor* is an Inſtrument ſo well known, and ſo eaſie to be made and procured, that I ſhall be very brief in the deſcription of it.

As

As it is usually made, it consists of two parts, a Scale and a Semicircle, but the Scale is no necessary part of it, but serving (if you be not otherwise provided) for other uses before mentioned in the case of Plain-Scales.

But the Semicircle is more essential, and it may be made of Brass or other Metal of any convenient size, as four Inches (more or less) for the straight side; this Semicircle being bounded as all others are by two Lines, the one right or straight, the other circular.

The right Line is divided in the precise middle by a Point which is in the Center, upon which the Circular boundary is drawn, and two other Arches concentrical with it.

This Center, when the Semicircle goes alone without the Scale, should be guarded with two little lips, on each side one, or a little loop, for more convenient turning of the Instrument about upon a Pin fixed in a Paper.

The arched or circular edge is divided into 180 Degrees or equal Parts numbred by Tens, upon the upper Concentrick Arch, from 0 to 180, and in the lower from 180 to 360. So that by applying the straight edge of the Protractor twice to any Line, (keeping the Center right upon a Pin fixed in the Line) that is, with the Semicircle first above it, and then below it, or contrarily, you may draw a whole Circle by the guidance of the Arch, or set out any number of Degrees, as will appear more plainly hereafter.

A *Line of Chords* is a Line divided into 90 unequal parts, whereof 60, and the Radius upon which

which the Circle was drawn, are equal, and the Diviſions upon that Line are equal to the next Extent in a right Line, of ſo many Degrees from the beginning of the Quadrant as anſwer thereunto.

When Lines of Chords are cut upon wood, 'tis both uſual and neceſſary that there be two Studs of Braſs, the one at the beginning, and the other at 60 Deg. with little holes for the feet of the Compaſſes, when you take the extent of the Radius, to preſerve the Line from being wounded by the Compaſſes; and being thus fenced, it will for need do the work of a Protractor, but not altogether ſo commodiouſly.

CHAP. XIV.

How to take the true Plot of a Field by the Plain-Table, upon the Paper that covers it, at one or more Stations.

THere are three ways or methods for doing this work, two more uſual and ordinary, the third more unuſual and extraordinary, though now pretty well known to moſt Surveyors, and in late Books publiſhed. The firſt performs the work by meaſuring every Line from the Inſtrument to every Angle, and is a very ſure ſubſtantial way where it can be done, as it ordinarily may in moſt Cloſes.

The ſecond doth it by meaſuring only the Station or the Diſtances, and is very quick, but not ſo ſure and exact as the other; yet if managed by a skilful Artiſt, that knows how to plant his Inſtrument, ſo as to avoid making acute Angles unneceſſarily, it will come near enough the matter in many caſes; as in meaſuring for Workmen, that take the Mowing or Reaping of Fields by the Acre, or when Tythes are let at a ſmall value *per* Acre, as in poor barren Pariſhes they uſually are.

The third is the way of Circuition or Perambulation, the Inſtrument being oft to be planted, and the Plot to be meaſured about, by which not only difficult Cloſes, but even the thickeſt Woods, yea Bogs, Meres, and Pools of Water, may be plotted, which by neither of the former methods can be performed.

In all theſe Methods, two things are to be performed:

1. At every Angle where there is no perſpicuous Mark already, as a Tree, Buſh, Stile, *&c.* one muſt be placed, as a white Paper, or ſuch like; or elſe ſome one muſt go from Angle to Angle, and remain there as your Mark to look at, till you bid him remove to another; only when Angles are very near you, this labour may be ſpared.

2. When ever you have occaſion to plant your Inſtrument more than once, (as it will often fall out in the firſt Method, and ever in the two latter) you muſt be ſure it ſtand juſt as it did the firſt time for ſituation, for which your Needle if well touched and hung, will be good

good direction, but is not thought ſufficient without back-ſight and fore-ſight, (by ſome practical Surveyors:) I ſhall therefore in due ſeaſon ſhew you that knack. Now for the firſt Method.

I. When you go about to plot your parcel of Land, find ſuch a place in it if poſſible, from whence you can ſee all the Angles, and in that place plant your Inſtrument covered with a ſheet of Paper, and turning it about till the Needle playing at liberty, hang over the Flower-de-luce, (or any other notable place that you make choice of) ſcrew it faſt. Then chooſing any convenient place in your Paper for a Center, and to repreſent your ſtation (or place where you fix your Inſtrument) make a prick with the ſmall point of your Compaſſes, to which prick applying the fiducial edge of the Index, (which is eaſily done if you keep the Point of the Compaſſes reſting in it) direct the Index by the ſight to all the Angles, and when through the ſlit or long ſight, you ſee the oppoſite Thread cut the Mark in the Angle, draw a neat dry Line along the fiducial edge to or from the Center: then meaſuring from the Inſtrument to every Angle, ſet the meaſure by a Scale and Compaſſes from the Center towards the Angle, upon the Line that points at it, making a prick in the Line where the Chains and Links (reckoned from the Center) do truly end: then ſhall Lines drawn by a ſtraight Rule from prick to prick, give you the perfect plot upon your paper, which you may divide (as hath been before ſhewed) into Trapezia's and Triangles, and

ſo find the true Content. To make which plain, mark this *Example*.

*Fi.*24 Suppoſe A B C D E F G H to be a Field; having planted my Plain-Table as before directed at a convenient advantage, ſo as to ſee all the Angles, (as at I) I make a prick to repreſent my ſtation in the little Circle ⊙ marked with I, upon which laying the fiducial edge of my Index, and directing the Sights to all the Angles, I draw dry Lines toward A, and all the reſt of the Angles in order from the Center; and then meaſuring upon the ground from the Inſtrument to the Angle A, I find it to be 3.45, which I ſet (by the help of my Scale) from the Center to the Point A, and ſo upon all the reſt according to their due meaſures, and then black Lines drawn from Point to Point, as from A to B, from B to C, *&c.* limit the true Figure of the Field according to the Scale I uſed, *viz.* of 400 in an Inch.

And now before I paſs to further varieties, let my Reader take notice of theſe following things.

1. From henceforth I ſhall forbear (for brevities ſake) to take any notice of the meaſures of Lines meaſured from the Inſtrument to the ſeveral Angles, having ſo often ſhewed how to meaſure by a Scale.

2. When I ſpeak of meaſuring from or to the Inſtrument, I always mean from or to that part of ground that is perpendicularly under the head of the Inſtrument, where you are to draw your Plot, which will ever be encloſed with the three Legs of your Staff.

3. That

3. That it's uſually the quickeſt way to meaſure firſt from the Inſtrument to the firſt Angle, and then back from the ſecond Angle to the Inſtrument, and ſo the reſt in order, ſtill one from the Inſtrument, and the other to it. *Fi.24*

4. It is no matter at all whether your Plain-Table be placed towards the middle of a Field, as was repreſented in this Figure, or at an Angle, as will appear anon.

5. In all workings by this Inſtrument, you muſt have a care that the Inſtrument be not moved out of its due place, till you have finiſhed the work of the preſent Station; for which purpoſe, caſt your eye now and then upon your Needle, obſerving whether it continue to hang directly over the ſame point you ſet it at when you began your work, and to rectifie your Inſtrument if you ſee cauſe. But becauſe all Tables have not Needles, and where Needles are, they are not accounted over-truſty, make uſe of the following help.

When you have planted your Inſtrument, and made a point or prick in your paper, repreſenting your ſtation, ſet the fiducial edge of your Index to it, and turning it ſoftly about till you find one remarkable thing or other upon one ſide of the Cloſe, and another on the oppoſite ſide as you look through the ſights of your Index (which we call fore-ſight and back-ſight) draw a remarkable Line with Ink, or rather with a Black-lead Pen quite over your Paper, which in this Figure is repreſented by the black-line K L: and then if you ſuſpect that by any accidental jog, or other caſualty, the Inſtru-

ment is any thing removed, you may eaſily try and rectifie it, by applying the fiducial edge to the ſame Line, and making uſe of fore-ſights and back-ſights again, upon the ſame Marks which you before obſerved upon the oppoſite ſides of the Cloſe.

But if there be no convenient place for the placing of your Inſtrument, whence you may ſee all the Angles of the Field, more ſtations muſt be made uſe of thus:

Fig.25 Let A B C D E F G H I K L M N be a Field whoſe Angles cannot be all ſeen from any one Angle, or other place in it: I plant my Inſtrument at the Angle A, and if it have a Needle, I mark what Degree of the Chord it cuts, or turn about the Table on the head of the Staff, till the Needle hang over ſome remarkable place; (as ſuppoſe the Flower-de-luce) and ſcrew it faſt; then ſetting up a Stick with a white paper or cloath on the head of it, where I intend my ſecond Station (as here at Q.) I make a prick or point in my paper, to ſignifie the point A upon the paper on the Table: to which point I apply the fiducial edge of the Index, and when I ſee the white at Q, ſo as looking through the ſlit, I ſee the thread cut it, I draw the Line O P quite through my Paper with a Black-lead Pen, and then keeping the fiducial edge ſtill upon the ſame point, and turning it round by degrees, I look at the Angles B C D E F L M N, ſtill drawing dry Lines with the points of my Compaſſes, and ſetting on the Meaſures from the Station A to every Angle meaſured to or from, as I did in the laſt Example. Then

Then I remove the Inſtrument to the place of my ſecond ſtation, having ſet up a Mark at A, and laying the fiducial edge to the Line O P, I turn about the Table upon the head of the Staff, till through the ſlit of the back-ſight, I ſee the thread cutting the Mark at A, and then ſcrew it faſt, ſo will my Needle, if a good one, hang directly over the ſame point that it did at the firſt ſtation; but however that be, fore-ſight and back-ſight will do the buſineſs; for which purpoſe it is good to take back-marks as well as fore-marks at every ſtation, as was taught in the Example of a ſingle ſtation, only taking notice that the back-mark when the Inſtrument is planted in an Angle, muſt needs be out of the Field; as ſuppoſe here at O. But to proceed. *Fig.* 25

Having meaſured the diſtance between the firſt and ſecond ſtation, and finding it to be 7.10, I ſet it upon the Line O P, from A to Q; where I make another point to repreſent the ſecond ſtation, and turning about my Index with the fiducial edge upon that point, and ſo looking through the ſights at the Angles G H I K, I draw Lines towards them on my Paper, and having meaſured between every one of thoſe four Angles, and the Inſtrument, I ſet thoſe Meaſures as I did the other, with my Scale and Compaſſes, from Q towards every Angle upon his proper Line: and then having drawn the black bounding-lines from A to B, from B to C, and ſo round about the Cloſe, the Protraction is finiſhed.

But here to make this Figure yet more advantageous

Fi.25 vantageous, let me (according to my uſual method) add ſome Advertiſements.

1. Sometimes a Station is ſo taken, that you may meaſure towards two Angles at once, (as here from Q to G and H) in which caſe you are to ſet down the Chains and Links where the firſt Angle falleth, but ſtill be proceeding to the further Angle, cauſing the remainder of the Chains at the fore-end to advance beyond the former Angle, ſo going on with whole Chains ſo far as you can, to which the odd Links at the end are to be added.

2. If at any of your Stations (as ſuppoſe A) you can ſee an Angle (for example E) to which you cannot meaſure in a direct Line without paſſing the boundaries of your parcel of Land given to be meaſured; you may notwithſtanding take in that Angle by a ſtraight meaſured Line, as I have done, provided it may be lawfully done without treſpaſs, and conveniently without troubleſom paſſing of Fences, otherwiſe it muſt be taken from another ſtation.

3. I here took one of my Stations at an Angle, and the other within the body of the Field, to ſhew the variety of working taught by other Authours, and that 'tis no great matter where you make your Stations, ſo you can ſee the Angles: elſe it had been full as convenient to have taken my firſt Station alſo within the body of the Field, as ſuppoſe at R.

4. Though this Figure repreſenteth to your eye onely two Stations, A and Q, your fancy may multiply them at pleaſure, for ſuppoſe the Angle H could not have been ſeen from A or

A or Q, how eaſie had it been to have ſet up a Mark at S, and then to have removed the Inſtrument thither, obſerving the ſame directions that were given at the removal from A to Q.

II. In the ſecond Method the Inſtrument is to be planted twice, or oftner as occaſion is, the Rules for removal of the Inſtrument, fore-ſight and back-ſight, and meaſuring the diſtance of Stations, being the ſame as formerly was taught: but inſtead of meaſuring to and from every Angle, we only view each Angle through the Sights from two Stations, having applied the fiducial edge to the Points repreſenting thoſe Stations, and having drawn Lines with the point of the Compaſſes, or a protracting Needle, the Interſections repreſent the Angles, from which the boundary-lines may be drawn, ſo is the Field protracted: Which that my Reader may underſtand, let him note theſe three Figures.

Here in theſe three Figures the Angles are marked Alphabetically, A B C D E F, *&c.* and the Stations by a point in a ſmall Circle numbred, 1, 2, or 1, 2, 3, according to their number and order. *Fi.* 26 27 28

The firſt of theſe Figures repreſents the Plotting of a Field at two Stations within it, from both which all the Angles may be ſeen. *Fi.* 26

The ſecond, performs the ſame work by two Stations taken without the Field, by which Art a Cloſe may be meaſured, though the preſent Poſſeſſour will not give us leave to come into it. *Fi* 27

The third, ſhews how the Work may be performed *Fi.* 28

Fig. 28 formed at three Stations or more, when two ſuch places cannot be found whence to view all the Angles; which laſt having more of difficulty than the two former, (though indeed not very much) and the Explanation of that will ſufficiently help to the underſtanding of them, I ſhall a little explicate the meaning of it in theſe particulars.

1. From the firſt Station taken according to former directions, I ſee the Angles A B C D F G K, and accordingly draw Lines upon my Paper towards them from the point repreſenting that Station, by the fiducial edge of my Index with the point of my Compaſſes.

2. Having removed my Inſtrument to the ſecond Station, (and in ſo doing, obſerved the Rules before given, touching ſuch removals) I thence ſee the Angles A B D E F G H I K, and draw Lines upon my Paper towards them from the point repreſenting the ſecond Station. And now viewing my Work, I find upon my Paper Interſections for the Angles A B D F G K, but only ſingle Lines towards the Angles C E H I: therefore

3. Removing the Inſtrument regularly as before, to a third Station, I thence ſee thoſe four Angles C E H I, and drawing Lines towards them, I have interſections for them alſo; ſo that having drawn the Lines A B, B C, *&c.* from one Interſection to another, I have the Field perfectly protracted. For theſe bounding-lines from Angle to Angle, do not only ſignifie the Boundaries of a peice of Land given to be meaſured, limitting the Figure or ſhape thereof, (and

(and are to that purpoſe given in this and all other Survey-books) but alſo are the true diſtance by a Scale from Angle to Angle for the Plot upon the Paper: I mean by the ſame Scale by which the ſtationary diſtances were laid down upon their own lines. And this holds true in all kinds of true plotting, whether in this Method or any other. *Fi.*28

III. The third Method is that of Circuition, and this hath ſeveral varietſes, according to theſe three following Caſes.

1. When the diſtance from Angle to Angle, (without any exception) is meaſured quite round the Plot, either within or without.

2. When the diſtance is taken only between ſome more notable Angles, and the Perpendiculars of the reſt meaſured as you paſs along their Baſes, within the Plot, proper for plain ſolid ground.

3. When the like is done without the Plot, as in the caſe of plotting thick Woods, Meres, Pools, Bogs, *&c.*

The firſt of theſe is very eaſie, conſiſting in nothing but planting the Inſtrument at every Angle (either within or without, as neceſſity and convenience determine it) obſerving the former directions for planting and removing the Inſtrument; and alſo for meaſuring the ſtationary-lines on the ground, and protracting them on the paper, as is manifeſt in this *Example.*

Let A B C D E F be a Park-Pond or Cloſe to be protracted, I firſt plant mine Inſtrument at A, and direct the ſights to a Mark in the Angle B, draw- *Fi.*29

*Fi.*29 B, drawing a dry Line from a convenient Point on my Paper towards B on the ground, and having measured by my Chain the distance A B, I set it by a Scale upon the Correspondent Line from A to B, drawing a black-line between them with Ink or a Lead-pen, the Extremities whereof are the Points A and B on my Paper, and the little pricked Line that goes beyond B, represents the remainder of the dry Line drawn at random (as to length) with the point of the Compasses.

Then setting up a Mark at A (if there was none before) I remove the Instrument to B, and laying the fiducial edge to the Line A B, I turn about the Instrument upon the Staff, till through the sight I perceive the thread cutting the Mark at A, and my Needle (if I have one) directly over the same point, that it was when it was planted at A, and so screw it fast.

My next work is to lay the fiducial edge to the point B, and direct the sights to C, drawing a dry Line toward it, and setting the distance B C measured by the Chain from B to C.

In this manner I proceed, surrounding the Close till I come at last to A, where I began, by planting the Instrument at every Angle, using the help of back-sight and my Needle, as I did at B, and then from the point representing my present station, directing the sight to the next Angle, as I did from B to C.

In the second Case, we do not plant the Instrument at every Angle, but at the more considerable, taking in the smaller by their Perpendiculars from the Base as we pass along, of

which

which this following Figure may be an Inſtance.

Let A B C D E F G H I K L be a Cloſe to be meaſured; by planting the Inſtrument only at A C F and K, I have the main ſubſtance of the Cloſe in the Trapezium A C F K, and for the five ſmall Triangles which muſt be added to the Trapezium, they may be eaſily protracted by the help of ſuch a little Square as was mentioned towards the latter end of the *Tenth Chapter*; for thereby finding at how many Chains and Links diſtance from A upon the ground, the Perpendicular B b falleth upon the Line A C; and having meaſured the length of that Perpendicular, and taken it between my Compaſs-points off my Scale, I erect a Perpendicular of that length at b, which is the point upon the Paper, where ſo many Chains and Links determine, as were meaſured upon the Ground, from the Angles A, to the place where the Perpendicular fell on A c, *viz.* at b. Juſt in the ſame manner I raiſe the other Perpendiculars, D d, G g, I i, and L l: and then by the help of the Perpendiculars, I draw (from and to the proper Angles) the Boundary-lines A B, B C, C D, D E, F G, G H, H I, I K, K L, L A; which together with the Line E F between the Angles E and F, give the true Plot of the Field in one large Trapezium, and five ſmall Triangles ready for caſting up. *Fig.* 30

The third caſe is ſo like the ſecond, that there needeth no new Direction concerning it, but to annex one plain Diagram; all the difference conſiſting in this, that becauſe we cannot go

within it, (being ſuppoſed to be ſome Pool, Bog, or thicket) we muſt of neceſſity go on the out-ſide, and conſequently all the Triangles made by inward Angles, and their Lines upon the meaſured Baſes, muſt be excluded by the boundary-lines from being any parts of the Plot, as here is manifeſt.

*Fi.*31 Suppoſing A B C D E F G H I K L M N O P, to be a great Pool, though here be fifteen Angles, I plant my Table only five times, *viz.* at A E F H and M, and upon the dry Lines A E, F H, H M, and M A, I raiſe their Perpendiculars in due places, (according to meaſure) and alſo of a right height: by which and my five ſtationary Angles, I draw the bounding-lines of the Plot, excluding all the Triangles as foreign to it, they being no reſemblances of any part of the Pool, but of Land adjacent.

Where note, 1. That both in this and the former Caſe, ſuch a little Square as I mentioned in the *Second Chapter*, will be very uſeful for ſpeedy raiſing of Perpendiculars; but where the Triangles are very ſmall, it needs not be uſed, nor the other mentioned *Chap.* 10.

2. That if by reaſon of troubleſom Bruſh-wood, Gorſe, or Bogs, *&c.* I could not have meaſured cloſe to the ſides E F, H I, or L M, it would be the ſame thing if I went parallel to them. And this is a ſhift that the practical Surveyor will oft be put to make uſe of, in other caſes as well as this.

CHAP.

CHAP. XV.

Concerning the Plotting of many Cloſes together, whether the Ground be even or uneven.

THough I deſign not ſo high in this *Manual*, as to make my Reader able to Survey Lordſhips and Foreſts, much leſs to draw Maps of Countries, but to meaſure a parcel of Land with truth and judgment; yet I would have him ſo expert, as not to be puzzled, if any ſhould deſire him to draw (as it were) a true Map of a Tenement or ſmall Demeſn, conſiſting of ſeveral Cloſes; for which purpoſe, let him that knows no better obſerve this Method.

Suppoſe A B C D E F G H I K L M to be *Fi.*32
a Tenement or ſmall Demeſn divided into fourteen Cloſes, to be meaſured and protracted according to their ſeveral ſhapes and ſituations. I firſt draw the Plot of the whole by the Method of Circuition, planting mine Inſtrument either at every Angle, or only the moſt conſiderable, either within or without, as I find moſt convenient. This being done, a Line from B to M gives the Triangle A B M for the firſt Cloſe: In the next place, I go round the ſecond Cloſe beginning at M, then to B, and ſo about *(cum ſole)* to M again: And then for the third Cloſe I plant my Table at C and go round to B, (the Line B C being protracted already) and ſo of all the reſt, ſtill obſerving which are com-

*Fi.*32 mon Lines belonging to ſeveral Cloſes (repreſenting the Fences) that I may avoid the trouble of meaſuring thoſe Lines oftner than once, and lay every part of every Cloſe in its due place; and that I be ſure to keep the Inſtrument throughout the whole Work to its true Poſition by Needle, fore-ſight and back-ſight.

There are I confeſs divers other ways of doing this Work, but none more ſure or plain, eſpecially if the ground be uneven; for in that caſe, if you protract according to the length of Lines meaſured from your ſtations to the Angles, you will put your Cloſes into unproportionable ſhapes, except you reduce Hypothenuſal Lines to Horizontal, by Inſtruments or otherwiſe (which is ſomewhat troubleſom:) and the like may be ſaid when you Plot with the Chain only. Indeed the method of meaſuring only the ſtationary diſtances were very proper for ſetting out the Figure of each particular Cloſe, provided the diſtance of the ſtations be large, and taken (if poſſible) upon pretty even ground, (which ſometimes may be done, though moſt of the Cloſe be uneven) and the Work ſo ordered, as not to make too acute Angles: but becauſe this requireth skill and care, I rather adviſe my young Artiſt to uſe the circling way, as ordinarily moſt commodious in my poor judgment, (but not prejudicing other men's that may differ from me in opinion) and where need requires let him obſerve the directions in the 17 *Chapter.*

☞ But which way ſoever you go to work, there is one very neceſſary Rule to be obſerved. If the

the Ground be uneven conſiderably, you muſt not give up the Content by meaſuring the Baſes and Perpendiculars of the Triangles on the Paper by your Scale; but you muſt meaſure the Lines correſpondent to them on the Ground, and caſt up the Content according to that meaſure. And if it be deſired that you ſhould adjoyn to your Plot (as is uſual) a Scale of Chains to meaſure diſtances by; you muſt either by making the Forms of Hills erect and reverſe, or ſome other Note in writing, mark out your uneven ground, leſt thoſe that try it by the Scale, judge your work erroneous: for though you make that Scale exactly corrreſpondent to that you protracted by (as you ought to do) the Hills and Dales in the ground truly meaſured, may make a conſiderable alteration. *Fi.*32

It is convenient when you plant your Table, that the Needle hang juſt over the North-point of the Compaſs under it in the Box; then may you by the Lines overthwart the Frame of the Table, eaſily draw two Lines quite through the Plot, cutting one another at right Angles, the one pointing at North and South, and the other at Eaſt and Weſt. And if your skill ſerve you to make the Two and thirty Points of the Compaſs upon the place where they interſect, and to draw the Forms of the Houſes, Woods, and other remarkable things upon the Demeſn, and the courſe of Brooks and Rivers running through it, it will add to your commendation.

And ſo it will alſo, if you take in ſuch parcels of Land bounding it, whether common or peculiar

culiar to other men, as will make your Plot to look handſomly, like a perfect Square or Oblong. But however that be, you muſt be ſure to protract truly all Lanes going into it, or through it, and all Cloſes of other mens mixed with it; and alſo all conſiderable Ponds, Ways, and Outlets, with the Names of the Cloſes and quality of the Ground, whether Meadow, Paſture, Arable, *&c.*

CHAP. XVI.

Concerning ſhifting of Paper.

IN ſuch work as that of the laſt *Chapter*, it may ſometimes fall out (through the multitude and largeneſs of Fields) that one ſheet will not hold your whole Plot, in which caſe you may help your ſelf by ſhifting Paper (as we call it) thus.

Fig. 33 Let A B C D repreſent our ſheet of paper that covereth the Table, upon which the Plot of the large piece of Land E F G H I K ſhould be drawn; having made my firſt ſtation at E, and the ſecond at F, I find my Paper will not receive the Line F G: but however I draw it ſo far as it will go to the edge of the paper, and planting my Table again at E, proceed in my Circuition the contrary way to K and I, where I find my ſelf again at a loſs for my Line I H, but draw it alſo to the edge of the Paper: Then with

with the Point of my Compaſſes ſtriking the Line P O, parallel to the edge of the Paper B C, and the Line Q O parallel to D C, and cutting P O in O, I throw aſide that paper for a while, covering the Inſtrument with a new one, which I mark with the figure (2) for my ſecond ſheet. *Fi.* 33

Upon which ſecond ſheet (the leading part whereof is repreſented by the three Lines meeting in the Angular points A and B) I draw P O parallel to A B the leading edge of the paper, and croſſing it at right Angles in the point O, by a parallel to B C, *viz.* the Line O R, being of the ſame diſtance from B C, that Q O in the former ſheet was from D C. Then with a Rule and a ſharp Pen-knife I cut off the end of the firſt ſheet at the Line P O, and applying the edge of it to the Line P O of the ſecond ſheet, ſo as it may touch that Line all along, and the Line Q O of the former, touch the Line O R in the latter, ſo as to make one Line with it: I draw the Lines P G, being the Remainder of the Line F G, and the Line O H being the remainder of the Line I H, and from their extremities the Line G H. And if the Plot required it, you might proceed on in the ſecond ſheet, and annex a third and a fourth, *&c.* as there is occaſion. *Fi.* 34

Theſe ſheets may be pieced together with Mouth-Glew or fine Paſte, applying the edge of the former (as you did upon the Table.) to the Line P O of the latter.

And note here once for all, that when I ſpeak of applying the edge of the paper to a Line, I mean

mean the preciſe edge cut by the Line P O: but when I ſpeak of drawing Lines to the edge of the Paper upon the Table, I hope none will think me ſo abſurd as to mean the edge that is couched under the Frame, but that my meaning is, that the Lines muſt be continued on the paper till they touch the Frame.

CHAP. XVII.

Concerning the Plotting of a Town-Field, where the ſeveral Lands, Butts, or Doles, are very crooked: With a Note concerning Hypothenuſal, or ſloaping Boundaries, common to this and the Fifteenth Chapter.

Fig. 35 SUppoſe A B C D E divided in the manner of a common Field, into ſeven Parts or Doles, belonging to ſeven ſeveral men: Firſt Plot the whole as before hath been taught, then meaſuring from A to B upon the Land: ſet one Note down (as you go along) at how many Chains or Links (or both) the Diviſion is between Dole and Dole, and accordingly mark them out by the help of Scale and Compaſſes in the Line A B on the Paper-plot. In the very ſame manner you muſt meaſure and mark out the Lines O C and E D; which being done, take the Paper off the Inſtrument, and laying it before

fore you on a Table, with the ſide A E towards you; the Compaſſes muſt be ſo opened and placed (as by a few tryals they may) that one foot reſting upon the Table, the other may paſs through the Points of Diviſion upon all the three Lines, *viz.* A B, O C, and E D, as in this Figure they do. *Fig.* 35

If the Content of any one or more of theſe Parts, Butts or Doles, be deſired without Plotting; it may be eaſily done without your Plain-Table thus: Take the breadth by your Chain at the head, middle, and lower end, and adding theſe Numbers together, the third part of their Sum is the equated breadth: by which multiplying the length meaſured down the ridge (or middle) the Product gives the Content.

But both in this caſe, and that mentioned in the 15th *Chapter*. The Figure of a Plot may be ſomewhat diſordered, not only by the unevenneſs of the ground within, for which I have given due caution already, (that being both the more common and more conſiderable caſe) but alſo by the great declivity of the Ground where the boundary-lines go, either of the whole Plot or particular parcels. For whereas in Plotting, every Line is preſumed to be Horizontal (or level) that it may paſs from Angle to Angle the ſhorteſt way, and that every part may be duly ſituated, and none thruſt another out of its right place: If it be not level, but falling down towards a Valley, or riſing up Hill, or compounded of both; a Line over ſuch Ground (though true for the meaſure, and for giving up the Content) will be falſe as to the Plot,

Plot, and therefore muſt be reduced to a level, and ſo taken off the Scale and protracted. For the doing of this there are ſeveral Inſtruments very proper, eſpecially Mr. *Rathbourne*'s Quadrant upon the head of his Peractor (though it were better to have a Semicircle than a Quadrant ſo placed) and divers others. But ſuppoſing my Country-friend to have no other but ſuch as I have already deſcribed; I ſhall ſhew him a plain eaſie way much uſed by practical Surveyors, eſpecially in *Ireland*, as ſome of themſelves have told me, being the very ſame that he may meet with in Mr. *Leybourn*'s Book, Intituled, *The Compleat Surveyor*; I mean the ſecond way by him diſcovered.

*Fi.*36 Suppoſe A B C to be part of an Hill falling within my Plot, my Boundary-line going crookedly from A to B, following the Surface of the Ground. To find the Horizontal Line (equal to A C;) I cauſe one to ſtand at the point A (the foot of the Hill) and to hold up the end of the Chain to a convenient height, and gently aſcending the Hill, I draw it level and make a mark where it toucheth the Hill, obſerving the number of Links betwixt mine Aſſiſtant's hand and that place, where he muſt take his ſecond ſtanding, and hold it up as before, and ſo I draw it out level again till it touch the place, where he muſt take his third ſtanding, noting the Links as before, and ſo proceed, till at laſt from his fifth ſtanding I draw the Chain level to the higheſt Point within my Plot, *viz.* the point B. And now as the pricked Lines of this Figure put together, are evidently equal

to

to the Line AC: So are the Links noted down at every Station, when ſummed up, equal to the Horizontal Line of that part of the Hill. *Fi.* 36

In the very ſame manner, only inverting the Order, you may find the Horizontal Line going down-hill, where that is moſt convenient: And if there be both Aſcents and Deſcents in one Line betwixt two Angles, the Horizontal Lines of both muſt be found and joyned together in Protraction.

All this concerning Declivities of riſing or falling Ground, is to be underſtood when they are conſiderable, and a very exact Plot required: for ſmall ones, eſpecially when much exactneſs is not expected, are not regardable.

CHAP.

CHAP. XVIII.

Concerning Plotting a piece of Ground by the Degrees upon the Frame of the Plain-Table ſeveral ways, and Protracting the ſame.

HItherto I have ſhewed the uſe of the Plain-Table as ſuch, and I think my Directions have been near as plain as the Inſtrument it ſelf: At which ſome quarrel for its over-plainneſs, expoſing the Art to proud ignorant people, who judging the reſt of the Surveyors work to be as eaſie as looking through ſights at a Mark, and drawing Lines by a Rule, are apt to undertake to uſe it, or ſlight the skill of ſuch as do. Others ſay, (and that truly) that for vaſt things, ſuch as Foreſts, Chaſes, &c. the Circumferentor is more proper: And every one muſt grant, that in wet Weather, either that a Peractor, a Theodolite, or Semicircle, muſt needs be better than the Plain-Table covered with the Paper which cannot endure wet. Hence it is that ſome Artiſts have to good purpoſe ſhewed how the Box ſcrewed to the Index, and that made to turn on the head of the three-legged Staff become a Circumferentor. And if theſe thus fixed be turned about upon the Center of the Table; they will (ſay ſome with good reaſon, Mr. *Leybourn* for one) perform the work of the Peractor, much better than the Peractor it ſelf. Others ſhew, (as I ſhall

ſhall briefly) that taking the Inſtrument as it is without the charge of further fitting it, or trouble of removing the Box, the Index turned upon the Center will by help of the Degrees on the Frame, perform the work of the Theodolite, to which the Semicircle is near of kin.

And though I might eaſily anſwer all theſe Objections, by ſaying the firſt is frivolous; ſuch fooliſh Arrogance being eaſily contemned or checkt (if worth the while) by putting ſuch conceited Fools upon the harder part of the work. The ſecond impertinent to our purpoſe, who deſign not to plot ſuch vaſt parcels of Land: And the third concerning only an extraordinary caſe, and that well provided for otherways, for ſure no man that hath not a Body of the ſame Metal with his Inſtruments, will ordinarily meaſure Land in continual Rain, (a ſudden ſhower may be fenced againſt by a cover:) And if any be ſo eager upon his Work, I have ſhewed how it may be done in the former Chapters of this Book, without planting any Inſtrument at all, by Chain, Scale, and Compaſſes alone: Yet I ſhall ſhew how the Plot of a Field may be taken by the Degrees on the Frame not every way that I could imagine, nor that I could tranſcribe, (for that would be tedious) but two ways only, whereof the one is proper for an ordinary Cloſe, where all the Angles may be ſeen from one Station within it, the other fitting any parcel of Land though much larger, what ever be the Figure of it.

For the former take this Example.

*Fi.*37 Let A B C D E repreſent the Figure of a Field to be plotted by the Plain-Table in rainy Weather: I put on the Frame without a Paper the graduated ſide upwards, and plant it in ſome convenient place, whence I can ſee all the Angles, as at O; then placing the Index upon the Table, ſo as the fiducial edge do at the ſame time go through the Center upon the Table, and the Lines upon the Frame of the Table cutting it perpendicularly at 360, (where the Degrees begin and end) and 180 (the exact half) I turn about the Table upon the Staff-head, till through the Sights (the Side marked with 180 being next mine eye) I ſee the Angle A, and then ſcrew it faſt, obſerving where my Needle cutteth, and by back-ſight cauſing a mark to be ſet up in the Line C D at the Point F, that the Inſtrument may be kept firm from moving (or be rectified if it be moved) during the Work. And now the Line A O F paſſing upon the Land from the Angle A, directly under the Sights of the Inſtrument to the Mark at F, is (as it were) the prime Diameter whence the Degrees of the Angles are to be numbred, and accordingly I mark the Angle A in my Table hereafter to be exemplified with 360 Degrees. But to proceed, turning my Index with the fiducial edge upon the Center, till I ſee the Thread cutting the Mark at B, the ſaid edge cuts upon the Frame at 76 Deg. 15 Min. which I note down for that Angle:

gle: The like work I do, turning the Sights to C D and E, (but not to F, for there is no Angle, but only a Mark in the Boundary) and I find mine Index to cut for every Angle as I have marked them within the pricked Circle of the laſt Figure, *viz.* 157 Deg. 35 Min. for C, 225 Deg. 20 Min. for D, and 278 Deg. and 50 Min. for E. *Fi.*37

Then I meaſure (or cauſe to be ſtrictly meaſured by others) the Diſtances betwixt the Place where the Inſtrument ſtands, and every Angle, and find them to be as I have ſet them upon the pricked Lines in the little Circle, *viz.* A 4 Chains 20 Links, B 4 Chains 3 Links, C 3 Ch. 84 Li. D 5 Ch. 35 Li. E 5 Cha. 6. L. And now my Table both for Lines and Angles is thus perfected, and the Work is ready for Protraction within Doors.

	D.	M.	C.	L.
A	36	20	4	20
B	76	15	4	03
C	157	35	3	84
D	225	20	5	35
E	278	50	5	06

Your judgment will eaſily inform you, that in ſuch weather we ſhall hardly ſtand to make our Table neat and formal, but any thing (how rude ſoever) that we can underſtand, doth the feat. A Welſh Sclate with a ſharp Style, (or for want thereof, a Black-lead Pen and a ſmooth end of an hard Board like a Trencher) is more convenient at ſuch a ſeaſon than Pen, Ink and Paper. But of all I would commend for expedition a Red-lead Pen, whereby you may mark out every Angle neatly with one touch upon the Table it ſelf, juſt where it toucheth the Frame, by help of the fiducial edge, and cloſe by it the

*Fi.*37 length of the Line from the Center to that Angle: All which may be eaſily cleared off by a wet Sponge or Cloath ſo ſoon as you have protracted. Or if through the ſpongineſs of the wood, the head of the Table (which we uſe to cover with Paper) were made a little reddiſh, what great harm were that? We are forced to do it more real wrong by the points of the Compaſſes in the ordinary way.

Now to protract our Obſervations: I draw upon a Paper the Line A F at adventures, ſo it be long enough, and ſtick a Pin in at pleaſure for the Center O, upon which I place the Center of the Protractor, ſo as the ſtraight ſide (or Diameter) of the Protractor may juſt lie upon the Line A F, the Limb or Arched-ſide being upwards towards B, by help whereof I make a prick or point on the Paper at 76 Deg. 15 Min. for B, and at 157 Deg. 35 Min. for C. according to the numbers neareſt to the Limb.

Then turning the Protractor about on the Pin with the Arch or Limb down towards D and E, till the Diameter lie again juſt upon the Line A F, I number downwards from the right hand towards the left, by that rank of Figures that are nearer to the Center, beginning 190, 200, *&c.* and over againſt the places where 225 Deg. 20 Min. and 278 Deg. 50 Min. fall, I prick the Paper at the ſide of the Limb, and through thoſe four points I draw ſo many ſeveral Lines, (having laid aſide the Protractor) upon which and alſo upon the Line A O, I mark out by Points the true meaſure of every Line (by a Scale) from the Center, and from thoſe

thoſe points drawing the Lines A B, B C, C D, *Fi.* 37
D E, and E A, I have the true Plot of the Field.

Where note by the way, that we eſtimate Minutes as well as we can both upon the Frame of the Plain-Table, and the Protractor, accounting half a Degree, 30 Minutes; a third part, 20; a fourth part, 15, *&c.* And though by this means it is impoſſible to avoid ſmall errours, 'tis eaſie to avoid ſenſible ones; and the like may be ſaid when we protract by a Line of Chords, of which I now come to treat.

Having proceeded in the Field as before, and made my Table for Lines and Angles, or done that which is equivalent by a Red-lead Pen, I draw the Line A F, and having extended my Compaſſes to the Radius (or 60 Degrees) on a Line of Chords, I ſet one Foot towards the middle of the Line A F, and with the other I deſcribe a Circle like that in this Figure of a five-angled Field, but much larger, according to the length of the Radius: Then extending the Compaſſes from the beginning of the Line to 76 Deg. 15 Min. I ſet one foot in the Interſection of the Circle by the Line A, and with the other foot make a Mark in the Circumference of the Circle upwards toward the right-hand, and through it draw the dry Line B O.

In the next place I ſubtract the Angle 76.15 from 157. 35, where the Index cut for the Angle C, and there reſteth 81 Deg. 20 Min. which I take off the Line as before, and ſet it upon the Circumference from the Interſection by B O, towards the end of the Diameter marked

*Fi.*37 with F, and through the Point where it falleth, draw the dry Line C O.

In like manner I ſubtract 157 Deg. 35 Min. from 225 Deg. 20 Min. and the difference is 67 Deg. 45 Min. which I ſet from the Interſection by the Line C O downwards paſt the prime Diameter A F, and through the point where it falleth, draw the Line D O.

Laſtly, Having ſubtracted 225 Deg. 20 Min. from 278 Deg. 50 Min. there reſteth 53 Deg. 30 Min. which muſt be ſet downward towards the left-hand from the Interſection by D O, and through the point where that falleth, I draw the Line E O. And now when I have ſet the particular Meaſures upon every Line, and drawn the Boundary-lines, as I muſt have done if I had uſed a Protractor, the Plot is finiſhed.

But for better aſſurance that I have done my Work well, I take the meaſure of the remaining Angle A O E upon its proper Arch, *viz.* from the Interſection of the Circumference by A F, to the Interſection by E O, and applying it to the Line of Chords, I find it to be 81 Deg. 10 Min. as it ought to be, for it ſhould be the Complement of 276 Deg. 50 Min. to 360, and ſo it is, 76. 15

And for further ſatisfaction, I ſum up the Degrees and Minutes of all the five Angles, which for plainneſs ſake I have noted in every one of them on the outſide of the Circle in the Figure ſo often referred to, and their ſum is 360, as it ought to be, and as here is evident.

81. 20	
67. 45	
53. 30	
81. 10	
360.00	

My

My Reader may now perhaps expect that I teach him how to take a Plot at two or more Stations, when all the Angles cannot be ſeen from one: But becauſe this is ſo eaſie from the grounds already laid, to any that is ingenious, and in part rendred unneceſſary by the Method preſently following, I ſhall content my ſelf to give this general hint.

When you have from one Station taken in all the Angles you can ſee from thence, and then are to remove to your ſecond ſtation, do juſt as you would do if the Table were covered with a Paper; only it is at your choice, whether you would guide your ſelf for back-ſight by a Line that may be rubbed off, drawn upon the Table it ſelf from the Center to the Degrees on the Frame along the fiducial edge, or by noting only what Degrees it cuts on either ſide of the Center, the edge paſſing through it, that by the help thereof and the Needle, the Inſtrument may be placed in the ſame Line and Situation as before, for taking in the reſt of the Angles, if it can be, if not, another Station muſt be taken after the ſame manner. But now to my ſecond Method;

Let A B C D E be the Figure of a Field to *Fi.* 38
be plotted, the Weather being bad: I ſend mine Aſſiſtants to find the length of every ſide, meaſuring it about, *cum ſole*, beginning at A, who return me ſuch an account of every ſide in Chains and Links, as I have noted them upon the Figure, and in the Table following, *viz.* A B 3 Chains, 73 Links, B C 4 Chains, 91 Links, *&c.* In the mean ſeaſon, I make haſt to find the Angles,

Angles, and without curioſity plant the Inſtrument at B, and laying the Index on the Center, I look at C, and find the Index cutting 10 Deg. 15 Min. and looking at A, it cuts 126 Deg. 45 Min. out of which if I ſubtract 10 Deg. 15 Min. there reſteth 116 Deg. 30 Min. for the Angle A: but becauſe I like not my Quarters ſo well as to ſubtract there, I ſet them down thus; B $\begin{matrix} A\ 126.45 \\ C\ 10.15 \end{matrix}$ the meaning whereof is, that B notes the Angle, and C A the Lines meeting there, cutting ſuch Degrees on the Frame, and the reaſon why I ſet A above, is for more ready ſubtracting afterwards: then removing to the Angle C, and thence looking at B and D, I find the Index to cut as here expreſſed, C $\begin{matrix} B\ 153.10 \\ D\ 15.40. \end{matrix}$

In like manner I find at D thus, D $\begin{matrix} C\ 96.05 \\ E\ 28.50 \end{matrix}$

At E thus, E $\begin{matrix} D\ 141.20 \\ A\ 11.45. \end{matrix}$

And laſtly, at A I find them thus, A $\begin{matrix} E\ 98.30 \\ B\ .9.20 \end{matrix}$

An.	*D.*	*M.*	*Sides*	*Ch.*	*L.*
A	89	10	A B	3	73
B	116	30	B C	4	91
C	137	30	C D	1	88
D	67	15	D E	6	64
E	129	35	E A	2	29

This being done, I haſt under Covert, and by Subtraction find 116 Deg. 30 M. for the Angle B. 137 Deg. 30 M. for C. 67 Deg. 15 Min. for D. 129 Deg. 35 Min. for E. and 89 Deg. 10 Min. for A, as you find them

them on the Figure, and in this Table, together with the length of the Lines. *Fi.*38

Note, that there is a way to find the Angles without Subtraction, if at every Station you lay the fiducial edge over the Center, and the Diviſions 180 and 360, turning about the head of the Inſtrument upon the Staff, till through the Sights you ſee one of the neighbouring Angles, for the Index turned upon the Center to the other Angle, will give you the quantity of the Angle you are at, but this exact planting at every Angle is more tedious than the other, and therefore not ſo fit for wet weather. But now to protract this Plot:

Firſt, By my Scale, Rule, and Compaſs, I draw the Line A B in length 3 Chains, 73 Links, ending at the point B: then laying the Center of my Protractor upon the Line A B, ſo as the Center of it be upon the point A, and that end of the Diameter from which the Numbers are reckoned on the Arch or Limb towards B, I make a Point for the Angle A at 89 Deg. 10 Min. by the guidance whereof and the point A, I draw the Line A E, which according to my Scale, muſt be 2 Chains, 29 Links.

In like manner placing the Diameter upon A E, juſt as it was upon A B, and the Center upon the point E, I mark out by the Limb (for the Angle E) 129 Deg. 20 Min. by which I draw the Line E D, 6 Chains, 64 Links.

In the next place, I bring the Center of the Protractor to the point D its Diameter, lying on the Line E D, and its Limb towards A, by which I prick out 67 Deg. 15 Min. for the Angle

at

*Fi.*38 at D. and draw the Line 1 Chain, 88 Links.

Laſtly, the Center being at C, and the Diameter upon the Line D C, in ſuch manner as before at other Angles, I prick out by the Limb or Arch 137 Deg. 30 Min. and draw the Line C B, for at B my Plot ſhould cloſe, and if rightly done, the Angle at B will be 116 Deg. 30 Min. and the ſide B C 4 Chains 91 Links, which by meaſure I find ſo to be.

But if I plot by a Line of Chords, I am not bound to this Order, but may go from A to B, and ſurround that way if I pleaſe, which I could not ſo well do with a Protractor, without reckoning my Numbers backward, yet it muſt be granted that a Line of Chords neither doth the work ſo quickly nor conveniently, for this is the way.

When I have drawn the Line A B of a right length, I ſet the Compaſſes to the Radius, and placing one foot of the Compaſſes in the point B, and with the other deſcribe an Arch of a competent length, beginning at that ſide of the Line A B, that is deſigned to be the inward-ſide, and upon this Arch, 116 Deg. 30 Min. muſt be ſet, but becauſe my Line of Chords gives me only 90, I ſet them firſt on from the Line A B, and then take off the remainder 26 Deg. 30 Min. I joyn them to the 90 upon the Arch, making a Point, through which the Line B muſt be drawn of a due length. In the like manner muſt I do at C and E, but the Angles at A and D need no ſuch piecing, being capable of being meaſured out by a Line of Chords at once.

Nor

Nor do your Angles only give you trouble in this kind of work, but oft-times your Lines will be found too ſhort to receive the touch of an Arch upon the Radius, eſpecially if the Line of Chords be large and your Scale little, and ſo it may often fall out when you uſe the Protractor upon ſuch ſhort lines as A E and C D of this laſt Figure: In which caſe a Rule muſt be applied to them, and they muſt be extended to a due length that the Arches may meet them without the Figure. And if thoſe Extenſions of Lines and deſcribing of Arches ſpoil the beauty of your Plot, the matter is not weighty; 'tis ſo eaſie to be retrieved, for if you lay it on a clean paper and prick through every Angle; Lines drawn between thoſe points will give you the Plot neat and perfect.

One thing more before I cloſe this long Chapter; the Artiſt ſometimes loſeth his labour of Protraction through ſome errour in the Field, ſo as his Plot will not cloſe: 'tis therefore good to know before we begin that work, whether it will or no; for which purpoſe if we take a Number leſs by two than the number of Angles in the Plot, and thereby multiply 180, that Product being found to be equal to all the Angles, the Plot will cloſe, and ſo it appears by our Plot in this preſent work: the Multiplier being 3, becauſe the Angles are 5, and the Multiplier muſt be two leſs than the number of Angles.

	Deg. Min.
180	89 . 10
3	116 . 30
	137 . 30
540	67 . 15
	129 . 35
	540

This kind of tryal is grounded upon two principles of *Euclid* and *Ramus* mentioned in the firſt and third Chapters of this Book, ſhewing that in all plain Triangles, all the Angles taken together, are equal to two right Angles, and that the ſides (conſequently the Angles alſo) of every triangulate Figure, are more by two than the Triangles of which it conſiſteth: But I think it not proper to be large in ſuch things whereof my young Artiſt is like to make but little uſe; for when all is done, I confeſs with Mr. *Wing* in his *Art of Surveying*, *Lib.* 6. *Chap.* 15. that this way of Plotting by the Degrees, is far more troubleſom, tedious, and liable to errour, than the other ways upon a ſheet of paper, and therefore not ordinarily to be uſed, but when neceſſity compelleth us.

CHAP.

CHAP. XIX.

Concerning taking inacceſſible Diſtances by the Plain-Table, and acceſſible Altitudes by the Protractor.

THe Subſtance of what is to be ſaid for the firſt of theſe, is gathered from the Inſtructions given for Plotting a Field, by meaſuring only the Stationary-diſtance; but to make the caſe more plain to an ordinary capacity: Suppoſe the Line A C to be the unknown breadth of a River, over which a Bridge or Boat is to be laid, and the General (that he may inform himſelf what ſtore of Boats and Planks is neceſſary to be brought down) commands me to tell him the true diſtance from A where he is at preſent, to C a little Boat-houſe on the other ſide the water. *Pi* 39

To ſatisfie his demand, I plant my Table covered with a paper at A, cauſing one to ſet me up a Mark at B at a good diſtance from me, along the Bank of the River (the further the better, if diſtance do not hinder ſight:) Then having choſen a Point to repreſent A, and laid the fiducial edge upon it, I direct my ſights towards C and B, and ſtrike lines towards them. Which done, I ſet up a Mark at A, and from thence meaſure to B, (6 Chains, 32 Links) and ſo plant my Inſtrument at B, laying the fiducial edge to the line A B, and turning about the head of the

*Fi.*39 Instrument upon the Staff, till through the sights I spy the Mark at A, and then screw it fast.

In the last place, I take 6 Chains 32 Links off my Scale, and set it on the Line A B, from A to B, and laying the fiducial edge to the point B, from thence direct the Sights to C, and draw the Line B C, meeting or cutting the Line A C in C: So shall the space A C measured on the Scale (*viz.* 8 Chains, 29 Links) be the distance desired; and because the Chain is 22 Yards long, if I multiply 8.29 by 22, the Product is 182 Yards and $\frac{18}{100}$ of a Yard, which by reduction is some little more than 13 Inches and $\frac{1}{5}$ of an Inch.

Now to take the height of a Tree, Tower, or Steeple by a Protractor, without any Arithmetical operation, hang a Plummet with a fine Silk Thread at the Center of it, and hold it stedfastly with that end to your Eye where the Numbers begin, then look streight along the Diameter, as if you were to shoot in a Cross-bow without a Sight (still removing backward and forward as there is occasion) till you see the top of the Tree, Tower, or Steeple, and the Thread at the same time fall upon 45 Degrees; so shall the distance from your Eye to the Tree, Tower, or Steeple, measured in an horizontal or level line, together with the height of your Eye above the bottom of it, be equal to the height thereof.

If either for convenience of sight, or any other reason, you think good to set the other end of the Diameter to your eye, then the Thread for the tryal aforesaid, must fall upon 135 Deg.

instead

inſtead of 45. Other ways of doing this work by this Inſtrument (or a Quadrant) with the help of Trigonometry, and by other Inſtruments, I forbear at preſent (till I write a ſecond Part) conſidering whoſe benefit is here intended.

CHAP. XX.

Of caſting up the Content of Land by a Table.

TO make up the number of my Chapters to an even ſcore, I ſhall add one at the deſire of my worthy Friend Mr. *S. L.* before mentioned (to whoſe Experience and Communicativeneſs I acknowledge my ſelf indebted for the notion of meaſuring crooked Lands or Doles at the middle and both ends, marking every where how the Diviſions fall, as is mentioned in the beginning of *Chapter* 17.) concerning the uſe of a *Table* borrowed out of the 46th *Chapter* of Mr. *Leybourns Compleat Surveyor*, ſecond *Edition*, *Page* 271, which with the uſe take as followeth.

Links	R.	P.
100000	4	00
90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	08
20000	0	32
10000	0	16
9375	0	15
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	09
5000	0	08
4375	0	07
3750	0	06
3125	0	05
2500	0	04
1875	0	03
1250	0	02
625	0	01

This *Table* conſiſts of three *Columns*, the firſt containing *Links*, the ſecond *Roods* (or *Quarters* of *Acres*) the third *Perches:* and the uſe of it is thus:

Suppoſe a Field to be 7 Chains and 25 Links long, and 5 Chains 50 Links broad, theſe by multiplication make 398750 (as here is evident) whereof five Figures being cut off towards the right-hand, the Figure 3 ſignifies Acres, and the reſt, *viz.* 98750 denote parts, and to reduce them into Roods and Perches, I firſt ſubtract from 98750 the greateſt number of Links in my *Table* that can be ſubtracted from it, *viz.* 90000 (and put down for it 3 Roods, 24 Perches which I find over againſt it in the annexed *Columns*) and the remain being 8750. I look in the *Table*, and find over againſt it 14 Perches, which by addition makes 3 Roods 38 Perches: So is the whole Content of the Field 3 Acres, 3 Roods, 38 Perches.

	7.25	
	5.50	
	36250	
	3625	
	3625	
3.	98750	

A	R	P
3	3	24
		14
3	3	38

But note here, that if the Remainder after the

the firſt Subtraction cannot be found in the *Table*, you may take the neareſt to it, ſo the errour will be but part of a Perch.

As for Example:

7. 35, being the half Perpendicular, and 9.23, the Baſe, give for their Product 6.7840$\frac{5}{8}$. The 6 ſignifies Acres, and from the reſt 70000 being ſubducted. (to which 2 Roods, 32 Perches anſwer) there reſteth 8405, which becauſe I cannot find in my *Table*, I take the neareſt, which is 8125, to which 13 Perches anſwer: So the whole Content of that Triangular Cloſe is 6 Acres, 3 Roods and 5 ſquare Perches, and a little better.

	A	R	P
	6	2	32
			13
	6	3	05

But *Manum de Tabulâ* ——

I am at preſent taking leave of my Country-man, and ſuppoſing he brought with him any competent ſtock of Natural Capacity, and ſo much Arithmetick as enabled him to Add, Subtract, Multiply, and Divide; I dare make him judge, after he hath as faithfully laboured to underſtand me, as I to be underſtood; whether I have not performed what I undertook in my Title or elſewhere.

FINIS.

AN APPENDIX CONTAINING XII. *PROBLEMS* TOUCHING Compound Intereſt & Anuuities.

ALSO

Two Mathematical Eſſays:

I. To make the Doctrine of Extraction of Roots by Logarithms more generally uſefull.

II. To Contract the Works of Fellowſhip and Alligation alternate, by a quick and eaſie Method.

Fruſtrà fit per plura quod fieri poteſt per pauciora.

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[illegible] Subtraction cannot be [illegible] in the Ta-
[illegible] the [illegible]
[illegible]

For Example:

[illegible] the [illegible]
[illegible]
[illegible] and [illegible]
[illegible]
[illegible] which became
[illegible] find in my [illegible] I take
[illegible] which A P P
[illegible] 6 2 [illegible]

[illegible]

[illegible] of my Coun-
[illegible] brought with him
[illegible] Capacity, and
[illegible] him to [illegible]
[illegible]
[illegible]
[illegible] to be understood [illegible]
[illegible] have not performed what I undertook in
[illegible]

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XII.

PROBLEMS

Touching Compound Intereſt and Annuities, expreſſed in Symbols, *to be reſolved by* Logarithms; *firſt preſented in Twelve ſhort Lines, to the Right Honourable the Lord* Delamer; *afterward Explained by the Inventor* Adam Martindale, *and by his Conſent preſented to the* Royal Society *by Mr.* Collins, *and now applied to pertinent Queſtions in a Practical way, to make them more plain and uſeful.*

THeſe *Problems* are diſtinguiſhed into three Ranks, whoſe *Symbols* are thus to be underſtood.

p Principal,
r Rate, *viz.* 1 *l.* with its Rate, } common to all the three Ranks.
t Time,

a Amouut or aggregate, proper to the firſt Rank.
ſ Sum of Principal and Arrearages, proper to the ſecond Rank.
d Difference of Principal and Worth, proper to the third Rank.

Their

Their Capitals ſtand for the Logarithms of the Numbers ſignified by the ſmall Symbolical Letters before mentioned.

D. ſignifies *Data*, *Q.* *Quæſita*, *Prob.* *Problem*, *Reſ.* *Reſolution.*

The firſt Rank, concerning Compound Intereſt for a ſingle Sum of Money.

1. Prob. $D.p,r,t.$ $Q.a?$ Reſ. $Rt+P=A$
2. Prob. $D.a,r,t.$ $Q.p?$ Reſ. $A-Rt=P$
3. Prob. $D.p,a,t.$ $Q.r?$ Reſ. $\frac{A-P}{t}=R$
4. Prob. $D.p,a,r.$ $Q.t?$ Reſ. $\frac{A-P}{R}=t$

Examples relating to the Four Problems in order.

Queſt. 1. *What will* 15 *l.* 10 *s. amount to in* 12 *years at* 6 per Cent. *Compound Intereſt?*

Anſw. 31 *l.* 3 *s.* 7 *d.* 3 *q.* as appears by the work.

Ruiz. of 1.06 = 0.25306
t = 12

050612
025306

Rt = 0.303672
P 15.5 = 1.190332

A 31.182 = 1.494004.

Queſt.

Queſt. 2. What is 31 *l.* 3 *s.* 7. 3 *q.* *due* *twelve years hence worth in ready money, abating after* 6 per Cent. *Compound Intereſt?*

Anſw. 15 *l.* 10 *s.* as here appears.

A 31.182 = 1.494004

R = 0.025306

t 12

050612

025306

Rt = 0.303672

$A - Rt = P$ = 15.5 = 1.190332

Queſt. 3. *At what Rate of Compound Intereſt will* 15 *l.* 10 *s. amount to* 31 *l.* 3 *s.* 7 *d.* 3 *q. in twelve years?*

Anſw. At 6 *per Cent.* as here.

A = 1.494004

P = 1.190382

0.303672

12) 6.303672 (0.025306 = 1.06

0

Queſt.

Quest. 4. *In what time will* 15 l. 10 s. *amount to* 31 l. 3 s. 7 d. 3 q. *at* 6 per Cent. *Compound Interest?*

Answ. In 12 years.

$A = 1.494004$
$P = 1.190332$

$R\ .025306)\ 0.303672\ (12 = t$
0

The second Rank, touching Annuities in arrear, grounded upon these two

Axioms.

1. The Annuity and Rate of Interest being given, the Principal correspondent to that Annuity is in effect given also, being easily found by the Rule of Three, thus;

As the Interest of any Principal (*ex gr.* of 1, 10, 100, 1000, *&c.*) *is to* that Principal: *So* the Annuity or Pension, *to* its Principal.

2. The Sum of the Principal and Arrearages of all the Payments being found, the Arrearages alone may be obtained by subtracting the Principal from that Sum.

1. Prob.

1 Prob. $D.p,r,t.$ $Q.s?$ Reſ. $Rt+P=S$
2 Prob. $D.s,r,t.$ $Q.p?$ Reſ. $S-Rt=P$
3 Prob. $D.p.s,t.$ $Q.r?$ Reſ. $\frac{S-P}{t}=R$
4 Prob. $D.p,s,r.$ $Q.t?$ Reſ. $\frac{S-P}{R}=t.$

Examples ſuited to the firſt and laſt of theſe Four Problems.

Queſt. 1. *What will* 33 l. per annum *amount to in* 14 *years at* 6 per Cent. *Compound Intereſt?*
Anſw. 693 *l.* 10 *s.* as here.

6 . 100 :: 33
33
300
300
6) 3300 (550
30
30
30
00

$R=0.025305$
$t=14$
0.101224
0.25306
0.354284

N $Rt=$

$Rt = 0.354284$
$P = 2.740363$

$S = 3.094647 = 1243|5$
$550|$
$693|5$

Queſt. 2. *In what time will* 33 l. per annum *riſe a ſtock of* 693 l. 10 s. *at* 6 per cent. *Compound Intereſt?*

Anſw. In 14 years, as here is evident.

6 . 100 :: 33 . 550

$550 + 693.5 = 1243.5 = s.$

$S = 3.094647$
$I = 2.740363$

25306) 0.354284 (14
25306
101224
101224
0

The third Rank of Problems touching Annuities anticipated or bought for a Sum in hand (or that which is equivalent) at compound Intereſt diſcounted, is bottomed upon the former of the two Axioms above mentioned, and this that followeth:

Axiom. If the difference of the Principal and Worth be once found, the Worth is eaſily obtained by ſubtracting that difference from the Principal,

cipal, which is ever greater, being the Worth of the Annuity at that Rate for ever.

1 Prob. $D. p, r, t. Q. d?$ Reſ. $P - Rt = D$
2 Prob. $D. d, r, t. Q. p?$ Reſ. $D + Rt = P$
3 Prob. $D. p, d, t. Q. r?$ Reſ. $\frac{P - D}{t} = R$
4 Prob. $D. p, d, r. Q. t?$ Reſ. $\frac{P - D}{R} = t.$

Examples fitted to the firſt and fourth of theſe Problems.

Queſt. 1. *What is* 17 l. 10 s, per annum *to continue for* 11 *years, worth in preſent Money, at* 8 per cent. *Compound Intereſt allowed to the Purchaſer?*

Anſw. 124 *l.* 18 s. 8 *d. as here is ſhewed.*

8 . 100 :: 17.5?

17.5

500
700
100

8) 1750.0 (218.75
0

$R = 0.033424$
11

033424
033424

$Rt = 0.367664$

$P = 2$

$P = 2.339948$
$Rt = 0.367664$
$D = 1.972284 = \frac{218|75}{93|817}$
$124|933$

Note, that if in ſtead of ſubducting Rt from P, I had turned Rt into the Arithmetical Complement 9632335, and added that to the P 2.339948, it would have done the ſame thing in a more convenient manner, (ſave that it is not ſo ſuited to the latter of the Problems) as here is evident, 10 being rejected (as in this caſe muſt ever be) from the *Index*.

0.033424
11
033424
033424
0.367664
9.632335
2.339948
1.972283

Queſt. 2. *In what time will* 17 l. 10 s. *pay off a Debt of* 124 l. 8 s. 8 d. *allowing the Creditor after* 8 per cent. *Compound Intereſt?*

Anſw. In 11 years, thus manifeſt:

8 . 100

8 . 100 :: 17·5 . 218·75=*p*
124·933,
93·817=*d*

P=2·339948
D=1·972284
R=33424) 0·367664 (11
0 .

Any one that underſtands the very Elements of *Algebra*, may contract theſe Twelve Problems into Three; for the Firſt of any Rank will by Reduction, Application, and Tranſpoſition, produce the reſt. Then the work of the ſecond Rank may be performed by the firſt, if one but underſtand, that in ſtead of the Penſion, he muſt take the Principal correſpondent to it, and work with it, till he have found the Amount, from which the Principal muſt be ſubducted when the Arrearages of an Annuity are ſought, and ſo proportionably in the reſt of the Problems.

My Noble Lord *Delamer* only noted down for his own uſe $Rt+P=A$, and $P-Rt=D$: by which and a ſmall Canon of Logarithms, he will quickly anſwer any Queſtion of this kind. But for the help of young Mathematicians, I have ſet them forth thus explicitely.

The only inconvenience of any importance that I can yet diſcover in this Method, is, that in both Ranks concerning Annuities, the ſecond and third Problems are rather for demonſtra-

tion of the other, and to compleat the Rounds than for any other great uſe, proceeding upon ſuch *Data* as are ſeldom given for finding the Penſions and Rate, (or if they were, the Work would be rendred uſeleſs) yet we are not left without ſufficient help to find out Penſions or Annuities by plain and proper *Data*. For in reference to the ſecond Rank, if the Arrearages (or Stock to be raiſed) with the rate of Intereſt and time, be given for finding out ſuch an Annuity, as at ſuch a rate, and in ſo much time, will raiſe a given Stock by its Arrearages; 'tis eaſily found thus:

Suppoſe an Annuity at pleaſure, and by the firſt Problem of the ſecond Rank, find out its Arrearages, : Then ſay by the Rule of Three,

As the Arrearages found, *to* the ſuppoſed Annuity: *So* the Arrearages given, *to* the Annuity required.

For clearing whereof, let this be the Example.

Queſt. *What Annuity will at* 6 per cent. *Comp. Intereſt, raiſe a Stock of* 693 l. 10 s. *in eleven years?*

Anſw. 33 l. *per annum*, as here is manifeſt.

Suppoſe 30 *l.* then the Work goeth on thus:

6 . 100 :: 30
30
6) 3000 (500
30
000

R. 0.

R. 0.025306
14
0.101224
0.25306
Rt = 0.354284
2.698970
S = 3.053254 = 1130|46
500|
630|46

630.46 . 30 :: 693.5?
30
630.46) 20805.0 (32.999
44046

Here we account 32.999 for 33, being ſo much within an inconſiderable trifle.

Likewiſe in relation to the third Rank, if the Worth (or Price) Rate and Time be given, and the Penſion to be purchaſed be required; find the worth of any ſuppoſed Penſion by the firſt, Problem of the third Rank, then the Proportion ſtands thus:

As the Worth found, *to* the ſuppoſed Annuity: *So* the Worth (or price) given, *to* the Penſion required; as in the Example following:

Queſt.

Queſt. *What Annuity to continue eleven years will be purchaſed for* 124 l. 18 s. 8 d. *at* 8 per cent. *Comp. Intereſt?*

Anſw. 17 *l.* 10 *s.* as here.

Suppoſe 12 *l.* Then

8 . 100 :: 12?
12

200
100

8) 1200 (150
8

40
40

00

$R = 0.033424$
11

0.033424
0.33424

$R^{11} = 0.367664$

$P = 2.176091$
$R^{11} = 0.367664$

1.808427 = 150|64|332
85|668

85.668

85.668 . 12 :: 124.933 ?
12
249866
124933
85.668) 1499.196 (17.5
60

If the Interval betwixt every two payments be leſs than a year, conſider what part it is; whether $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{1}{13}$ $\frac{1}{52}$ $\frac{1}{365}$, *&c.* And by the Denominator of the Fraction ſignifying that part, divide the Logarithm of 1 *l.* and its Rate of Intereſt, (uſually called *the Logarithm of the Rate*) then the abſolute Number anſwering to that Quotient, being made leſs by an Unite, will be a new Diviſor, whereby dividing the given Penſion ſo payable, half yearly, quarterly, *&c.* the correſpondent Principal will appear in the Quotient, with which you may proceed, as if the Payments were ſo many yearly ones; as this next Example ſhews.

Queſt. 1. *What is a Quarterly Rent of* 3 l. 15 s. *to continue Thirteen Years and one Quarter, worth in ready Money at* 6 per cent. *Compound Intereſt?*

Anſw. 137 *l.* 10 *s.* 1 *d.* as appears here.

$R=4$) 0.025306
0.006326=1 | 01467
−1 |
| 01467

.01467)

.01467) 3.75000 (255.623
1059

$\frac{1}{4}R$=0.006326
Number of Quarters 53
0.018978
0.31630
$\frac{1}{4}Rt$=0.335278
Ar.C. 9.664721
P=2.407600
2.072321=255|623
118|119
137|504

Whereas I multiplied the Fourth part of the *Log.* of the *Rate*, viz. 0.006326 by 53 (the Number of Quarters) I might as well have multiplied the whole *Log.* of the *Rate*, viz. 0.025306, by 13.251, *i. e.* 13 years and 1 quarter, and it would have been full as exact, or rather more, (two places being cut off from the *Logarithm* so multiplied towards the right hand:) I say rather more exact, because there remained 2 when I quartered the *Log.* of the *Rate*.

Queſt. 2.

Queſt. 2. *What is* 5 s. per *Week to continue* 7 *years and three quarters worth in ready Money at* 6 per cent. *Compound Intereſt?*
Anſw. 75 *l.* 17 *s.* 8 *d.* 2 *q.*

52) 0.033424 (0.000642 = .00148
40

.00148) .25000 (168.918
136

R = 0.033424
t = 7.75
0 167120
2 33968
2 33968
Rt = 0.259036.00
Ar.C. 9.740963
P = 2.227675
1.968638 = 93|033 168|918
75|885

Here I multiplied (not by the Number of Weeks) but by the Number of Years and Quarters, cutting off two Cyphers from the Product. And I alſo took the Arithmetical Complement of 0.259036, *viz.* 9.740963, and added it to the Log. of the Principal in ſtead of ſubtracting the Log. 0.259036 from it.

Laſtly, As to the Rate of Intereſt, according to the ſenſe of Dr. *Newton*, Mr. *Dary*, and others,

thers, I ſee not why this Method may not be made uſe of as well as any other. This univerſal Rule being firſt underſtood: *In Queſtions* belonging to the *Second Rank*, the *greater Rate of Intereſt*, the *greater Arrearages*; and the *leſs Rate*, the *leſs Arrearages*. But in *Queſtions* belonging to the *third Rank*, the *greater the Rate of Intereſt*, the *leſs* the *preſent Worth* (or price) and the *leſs* the *Rate of Intereſt*, the *greater* the *Worth*, *& vice verſâ*. I ſhall ſhew this in anſwering the Queſtion following, concerning a yearly payment of 1 *l.* becauſe that by bare Multiplication may eaſily be applied to any other.

A Field worth 1 *l. per annum* clearly, is offered to be let 11 years for 15 *l.* to be paid at the end of thoſe years: What is the Rate of Intereſt demanded for the ſeveral payments?

Anſw. Firſt, I ſuppoſe 6 *per cent.* and trying it by the firſt Problem of the ſecond Rank, I find the Arrearages to be but 14 *l.* 19 *s.* 5 *d.* which is 6 *d.* 3 *q.* too little.

Then trying what the Arrearages will amount to at 6.2 (or 6 *l.* 4 *s.*) *per cent.* I find 15.13, or 15 *l.* 2 *s.* 7 *d.* 1 *q.* Perceiving then I have over-ſhot my Mark, I make my third tryal at 6.1 (or 6 *l.* 2 *s.*) *per cent.* and the Reſult is 15.05, or 15 *l.* 1 *s.* which is pretty near, being but 1 *s.* above the truth.

But that I may yet bring it nearer, I firſt ſubtract the Reſult at 6 *per cent.* which is 14.971 from the Reſult of the Arrearages at 6.1, *viz.* 15.05 (becauſe theſe are the Reſults that come neareſt to the truth) and their difference is .079

2. I ſub-

2. I ſubtract the Rate 6 from the Rate 6.1, and there reſteth .1

3. I ſubtract the neareſt Reſult to 15, *viz.* 14.971, from 15 the given Arrearages, and their difference is .029. Then working by the Rule of Three, I find (as followeth) .0367

.079 . 1 :: .079
1

.079) .0290 (.0367
7

Which .0367 being added to 6, gives 6.0367, or 6*l.* 0*s.* 8*d.* 3*q.* *per cent.* Compound Intereſt. And that this Rate exactly fitteth, is plain by the following Work.

6.0367 . 100 :: 1
1

6.0367) 100.0000 (16.565
20245

₹ 0.025456
11

0.025456
0.25456

0.280016
1.219191

1.499207 = 31|565
16|565

15|000

But perhaps it will be ſaid, that though in this and all other matters foregoing, I agree well enough with the learned Authors before mentioned, and others that have wri of theſe Subjects; yet in my Country-Almanack for the Year 1677. The Fourth *Problem* agrees neither with them, nor what I have here written, but apparently claſheth with them.

I confeſs this Charge had been very juſt, had that *Problem* been deſigned for the *ſame* end, and produced a *different* effect: (for in ſuch a caſe to differ is to contradict.) But foraſmuch as in my very entrance upon it, I began thus. *There is a* Problem *in ſome Learned mens Works, ſeemingly of the ſame importance with this, but indeed much different both in the Deſign and Effect:* And after in anſwering Objections ſhow wherein the Deſign differs; I cannot but admire the Undertakings of that able Artiſt, who was ſo much at leiſure as to prove with a great deal of Pomp, that mine will not attain the end for which I declared it was never deſigned, as clearly as words can utter it. A Sword may be a very good one, yet a very bad Inſtrument to fell Trees with. But I hear he is dead, and I ſhall rather lament the loſs of him, and divers other famous Mathematicians (which Death hath of late deprived us of) than unneceſſarily to expoſe the impertinence of his Paper to the Publick View; or ſo much as name him to the prejudice of his Memory. But from henceforth I expect from all ingenuous perſons, that they neither take for granted what I profeſſedly deny and diſprove; nor urge mine Objections againſt my ſelf,

ſelf, without taking notice of mine Anſwers.

For underſtanding of what follows, it were very convenient to have inſerted herein the whole Diſcourſe in the Country-Almanack about this buſineſs; but becauſe this little Paper cannot afford it room, I ſhall only point briefly at the Deſign and Management; and anſwering all the Objections I ever met with of any ſeeming Importance, make things as clear as I can in ſo narrow a compaſs. The Deſign was to diſcuſs in plain language (ſuitable to Country-mens underſtanding) this practical Queſtion, wherein they are oft concerned; *viz.* Whether it be more advantage for the Lender to receive for 100 *l.* in hand, Compound Intereſt at 8 *per cent. viz.* 71 *l.* 7 *s.* 7 *d.* 3 *q.* at the end of ſeven years above the ſtock of 100 *l.* ſuppoſing it can be legally aſſured (as in *Ireland* it may, and in *England* by equivalence in Goods or Lands, without mentioning Intereſt.) Or to enjoy a Farm for ſeven years in conſideration of 100 *l.* that will clear him juſt 20 *l. per annum*?

My Anſwer was, that the former was the better Bargain (and conſequently the higher Rate of Intereſt for the whole Stock during the whole term) for which I gave in theſe grounds under the feigned perſons of *A* and *B*.

1. *A* is to receive (as is ſaid) 171 *l.* 7 *s.* 7 *d.* 3 *q.* and if *B*, who takes the Farm, receive not equivalent, his rate of Intereſt is lower.

2. *B* receives only 140 *l.* by 20 *l. per annum*; only he hath the advantage to improve the ſeveral payments from the time they grow due, to the end of the term.

3. Theſe muſt be computed at ſome certain rate of Compound Intereſt: For to compute them at no rate of Intereſt, or at Simple Intereſt, or one taken up arbitrarily, will not ſute the caſe.

4. The Rate tolerated by the Statute, *viz.* 6 *per cent*, under which none will take, and above which none dare expreſly bind any to give, and at which any reſponſible man may be fitted, is to be preferred before any other.

5. At this Rate the 7 payments will amount but to 167 *l.* 17 *s.* 6 *d.* *q.*

Of theſe 5 Pillars, the 1, 2, and 5th, were never attempted to be ſhaken by any that I know of: What aſſaults have been made againſt the other two, or deſign of the whole Fabrick, I ſhall briefly conſider; and I find them (beſides thoſe anſwered when I firſt publiſhed that Problem, to be theſe three:

1. *Ob.* The Queſtion is not at all, how the Receiver improves his Payments, but what Improvements was made in the Debtors hands, and the ſame Rate carried on?

Anſ. We are agreed that the Rate is not to be computed according to what the Uſurer actually makes of it. If he gives away, loſes, or lends freely all the ſeveral Payments: Or if he make new Bargains more oppreſſive than the Original one; the true value of the Loan of ſo much Money for ſuch terms, is one and the ſame. But I utterly deny that it is to be reckoned, as if the ſame Rate muſt be carried on, and that for unanſwerable Reaſons (as I ſuppoſe them to be) laid down in my former diſcourſe, or eaſily colligible from it.

1. The

1. The Laws of our Nation prohibit upon ſevere Penalties, the taking of more than 6 *per cent.* therefore that is the utmoſt of the legal worth, which he that exceeds, runs (as I take it) the hazard of all, and a great fine.

2. If he can evade the penalty, no reſponſible man needs to give more than 6 *per cent.* and few are willing to give more than they need.

3. If he get over both theſe blocks, and make new oppreſſive Bargains, this is nothing to the Queſtion as I propounded it; for that was, whether *A.* or *B.* received the higher Intereſt for his Money, *by vertue of the above-mentioned Bargains*, not by vertue of occaſional after Bargains. When the Debtor hath paid in a years Penſion, he hath done with it; and if he have it not ready, he may take it up at ordinary Intereſt, and the proportion is broken off for ſo much.

2. *Object.* If we regard Laws in the caſe, what need we any Rules concerning Compound Intereſt, ſeeing the Laws of the Land allow only ſimple Intereſt.

Anſ. 1. The Laws are not againſt Compound Intereſt, as it may be managed: that is, the Uſurer may receive his ſimple Intereſt at the years end, and put out that as a new Stock, and ſo undeterminately from time to time; or if you will call this ſimple Intereſt, it's the ſame to the purſe.

2. I am told they have ways in *London* for putting out very ſmall Sums to mean Traders upon ſufficient Security; and then it is both legal and practical, though I ſay not how lawful be-

fore God, eſpecially as I hear ſome uſe it.

3. *Object.* It ought to be a Queſtion only of Art, without dependance upon Laws and Uſages; it being the nature of Art in theſe kind of Queſtions, not to ſhew ſo much what ought to be done, as what is really done.

Anſ. 1. It is one Queſtion, what is the nature of Art, and another how far Art is concerned: that is, whether nothing elſe ſave the Rules of Art be regardable in the caſe; which I deny, becauſe Laws and Uſages have a great influence upon it, hindring the continuance of the proportion.

2. I conſider not the Laws as pinching the Uſurers Conſcience (as to what is lawful) but as tying up his hands, and ſo obſtructing the proportionable increaſe of his gain.

3. Whatſoever may be ſaid of Art (as Art in the ſtrict notion of it) the Artiſt muſt not be ſuch a Slave to the Rules of it, as not to allow for unavoidable Obſtructions and Irregularities. I hope I may be allowed to tell my Scholars that learn Navigation, that though the direct courſe from one Port to another be upon ſuch a Point of the Compaſs; yet other Courſes muſt be ſteered ſometimes in regard of Rocks, Shallows, crooked Channels, Currents, Trade-winds, convenience of Harbour, and freſh Supplies: Or to avoid Pirats, Enemies, Forts, and Places where great Cuſtoms or Payments will be exacted; and many ſuch things which the experienced Sea-man is better acquainted with, than I with their Names. And to ſay to ſuch as learn Merchant-Accompts, that though it were more Arti-

Artificial and Rational, that Rebates of Intereſt for Money paid before it be due, ſhould be computed at Compound Intereſt (as certainly it is:) yet foraſmuch as it is uſual with Merchants to allow no more than ſimple Intereſt (as appears by printed Accompt-Books) they muſt ſubmit to the Laws of the great Tyrant *Cuſtom*.

4. An *Artiſt* hath not the ſame liberty of ſuppoſition in anſwering Queſtions (eſpecially ſuch as are real and practical) that he hath in propoſing Queſtions, or receiving them from others, when tryal of skill is only or chiefly intended. I may without abſurdity in this latter caſe demand or find the Amount or preſent worth of any ſingle Sum of Money, or yearly Penſion for any terms propounded at any rate of Intereſt given, though ſuch as is never likely to come in practice: (*ex. gr.* at 2, 3, 14, or 20 *per cent.*) But if I be put by the nature of my Work, to compute the Amount or preſent worth of Payments, no rate of Intereſt being named, I ought prudently to weigh all Circumſtances, and pitch upon that which is poſſible and rational. And this is not a work of *Art*, but of *diſcreet Judgment*, (wherein great reſpect is to be had to penal Laws, Uſages, and other Obſtructions) after which *Art* takes its own *Province*, in computing after ſuch a Rate reſolved upon.

To make all this plainer (if poſſible) than I made it at the firſt publication; Let us for once ſuppoſe a rare caſe, *viz.* That *B* a *Mathematician* turns Uſurer, and for 100 *l.* ready Money, takes a Farm for 7 years, that he lets to another for 20 *l. per annum* clearly, knowing before

fore hand he could ſo let it: and *A* his familiar Friend thus accoſts him; *I wonder to hear you are grown ſuch an Extortioner, as to receive* 20 l. per annum, *ſeven years together for* 100 l. *B* anſwers, Before you find fault with the Mote in mine Eye, take the Beam out of your own: You have bought the *Reverſion* of a piece of Land after ſeven years for 100 *l.* for which I will *bona fide* give you 172 *l.* at the time of your Entrance upon it; which I find by the Rules of mine Art to be 12 *s.* 4 *d.* 1 *q.* above compound Intereſt at 8 *per cent.*

A replies, Learned men, ſay ſuch a Bargain as yours will clear 9 *l.* 3 *s.* 4 *d. per cent.* and better. *B* rejoyns, 'Tis true they do ſo, but then they ſuppoſe the ſame Rate to be continued, (for none will ſay, that if the Payments lie by unimproved, or be let out at an under Rate, that in ſuch caſes that Rate of Intereſt can be anſwered) and this Suppoſition is really impracticable; or (to ſay the leaſt) that which a rational man cannot depend upon. I did accidentally meet with this Bargain from the hands of a weak man, which yet I durſt not have accepted (the Laws are ſo ſtrict) if it had been a Rent-charge in Money. And will you undertake to find me Fools that ſhall at every years end, take off the ſeveral Payments, yea and all the increaſe of them as they grow due upon the ſame terms that I put out the 100 *l.* upon, and to let me Land for it? If you think you can, I will make you a great Bargain, (becauſe I know you to be a punctual reſponſible man) you ſhall receive the firſt yearly Payment, and

and at the end of the ſecond year, pay me ſimple Intereſt for it at *6 per cent.* as the Statute allows, and I will inſtantly return it you, together with the ſecond 20*l.* which we will joyn into one Sum, and you ſhall take it at the ſame rate: and thus we will do every year to the end of the term, ſtill adding that years growth to what was before, and ſo keep up all the Accounts to *6 per cent.* and if you can make any more profitable uſe of it, much good may it do you, and I will heartily thank you to boot, for helping me ſo readily to place out my ſmall parcels without loſs. But if you dare not do this, let ingenuity mollifie your charge, and I ſhall not quarrel with you about your rate of 9*l.* 3*s.* 4*d.* which perhaps you borrowed (or ſome one for you:) from Dr. *Newton*, his *Trigonometria Britannica*, *P.*37. for I acknowledge it is that and ſomewhat more, though not much; but I ſay (which I deſire you to obſerve) that I receive no compound Intereſt after that rate, but only ſimple: and that not for the whole Sum during the whole term (as you do compound Intereſt for all your Stock) but only for the whole 100*l.* for the firſt year: for you know that if at the end of that year 20*l.* be ſubtracted from 109*l.* 3*s.* 4*d.* all the Intereſt is paid, and ſo much of the Stock as brings it down to 89*l.* 3*s.* 4*d.* This 89*l.* 3*s.* 4*d.* being computed at the ſame rate of Intereſt, 20*l.* abated from it, clears the Intereſt again, and brings down the Stock much lower: and ſo year by year the Stock is dwindled away, till at laſt the ſeventh payment (if the Rate were abſolutely exact, as it is near)

clears

clears off all Stock and Intereſt. And this is all I receive, ſave only the benefit of the ſeveral yearly payments, which I offer you (or any other ſolvent man) at *6 per cent.* as aforeſaid: At which rate (abating not a Farthing for loſs of time, but ſuppoſing good places ready for every parcel as it becomes due) it will but amount to 167*l.* 17*s.* 6*d.* *q.* which being the whole Aggregate of Stock and Intereſt, if we ſubſtract from its *Logarithm*, the *Log.* of 100, and divide their difference by 7, as we are directed by Mr. *Wingate*'s Λογαριθμοτεχνία, *Chap.* 5. *Prob.* 13. the rate of Intereſt will appear to be 7*l.* 13*s.* 7*d.* 2*q.*

I profeſs freely, Sir, I cannot withſtand *A*'s Arguments; but if any other can, I ſhall neither envy his happineſs, nor deſpiſe, (if fairly and ingenuouſly offered) his *Animadverſions*; in that caſe you ſhall command a Sheet from me at any time, in conſideration of any thing ſo objected; but for ſuch injurious and paſſionate diſcourſes as are apter to provoke than convince I confeſs 'tis ſuch a piece of drudgery to anſwer them, that (if I could not eaſe my ſelf by contempt) I ſhould think it hard meaſure that my ſilence ſhould be interpreted as their Victory.

Sir, You will pardon this *Prolix Appendix*, when 'tis conſidered that it is not intended for ſuch as your ſelf, but for ſuch to whom nothing can be too plain, by

SIR,

Your Friend and Servant,

Adam Martindale.

A Compendious Method for working out many Concluſions in Arithmetick, *wherein the* Rule of Three *is often repeated; long ſince invented, practiſed and taught by the* Author, *but never till now publiſhed.*

IN all Books of *Arithmetick* that I have had the happineſs to peruſe, the works of both Rules of Fellowſhip and Alligation alternate are performed by the *Rule of Three*, ſo oft to be repeated, as there are particular proportionable parts to be found; which proving intolerably tedious, when the Partners in Fellowſhip, or the parcels in Alligation are many, cauſed me to think of this following courſe, every way as plain and uſeful, and much more compendious, which to be ſhort is this:

The Rule.

Divide that which is uſually made the ſecond Number in the *Rule of Three*, by that which is as uſually made the firſt and the common Diviſor; That *Quotient* multiplied ſeverally by the reſpective third Numbers, gives the particular reſults. But here it will be neceſſary if you uſe *Decimal*

Diviſion (which I judge moſt convenient) that you continue your *Diviſion* till either nothing remain, or you have ſix places of *Decimals* in the *Quotient*, accounting *Cyphers* (if there be any) into the number. And if upon the *Addition* of your Parcels, the Total amount not juſt to the Sum expected, but an Unite ſhort in the Integers, and three or more Figures of 9 immediately following, take it for exact, for ſo it is within a matter of nothing.

All this will be plain by three *Examples* whereof the firſt ſhall be wrought (for plainesſ) both in the old and in this newer way; the other two (for brevity) this latter way only.

1. *Example in Fellowſhip without time.*

A, B, C } Put into the Common Stock { 15, 24, 33 } The Sum whereof is 72 *l.* and their gain 63 *l.* what is each Mans part?

72

The Reſolution in the old Method is thus.

A 72 . 63 :: 15

15

315

63

72) 945 (13.125

0

Or 13 *l.* 2 *s.* 6 *d.*

B 72 . 63 :: 24
24
252
126
72) 1512 (21 *l.*
0

C 72 . 63 :: 33
33
189
189
72) 2079 (28.875
0 Or 28 *l.* 17 *s.* 6 *d.*

13.125 13-02-06
21. 21-00-00
28.875 Or thus, 28-17-06
63.000 63-00-00

In the new way the Work is thus done.

72) 63.0 (.875 *A.*875
0 15
4375
875
13.125

B .875	*C* .875
24	33
3500	2625
1750	2625
21.000	28.875

I hope by this time mine intelligent *Reader* is aware, that if the Partners had been many (as in Voyages and Adventures it oft falls out) the difference betwixt the two Methods would have been yet more ſignally conſpicuous; for one ſingle Diviſion ſufficeth, be the Partners never ſo many: Though I confeſs there is yet a nearer way by the help of *Logarithms*; of which I ſhall preſent this Specimen.

63 = 1.79934055	.875 = 1.94201805
72 = 1.85732250	15 = 1.17609126
.875 = 1.94201805	13.125 = 1.11810931
.875 = 1.94201805	.875 = 1.94201805
24 = 1.38021124	33 = 1.51851394
21 = 1.32222929	28.875 = 1.46053199

The ſecond Example *in Fellowſhip with time.*

A, *B*, *C* laid in {43, 52, 63} for {8, 5, 7} Months {They gained 73 *l.* *Q.* each Mans part?}

The

The Resolution by this new Method is thus.

A	*B*	*C*	
43	52	63	344
8	5	7	260
344	260	441	441
			1045

1045) 73.00 (.069856
480

A. .069856
344

279424
279424
209568

l. s. d. q.
24.030464 *or* 24—0—7—2

B.069856
260

4191360
139712

18.162560
l. s. d. q.
or 18—3—3—0

C.069856
441

069856
279424
279424

30.806496
l. s. d. q.
or 30-16-1-2

Proof

Proof	or thus			
	l.	*s.*	*d.*	*q.*
24.030464	24—	00—	7—	2
18.162560	18—	03—	3—	0
30.806496	30—	16—	1—	2
72.999520	73—	00—	0—	0

The third Example in Alligation alternate.

Suppose a mixture of Wine of 119 Quarts be required, that must be made up of these several prices 7*d.* 8*d.* 14*d.* and 15*d.* so as the whole may be afforded at 12 *per* Quart, the parts may be found out in this method (without decimals) thus.

Having linked 8 to 14, and 7 to 15, and Counterchanged their differences from the Common price 12*d.* I find the Sum of their differences to be 14, by which dividing 119 the quotient is $8\frac{7}{14}$ or $8\frac{1}{2}$, which for convenience of Multiplication we shall change into the improper Fraction $\frac{17}{2}$ so the Resolution will be thus.

8	2	
14	4	
7	3	
15	5	
		14

Quarts.

$\frac{17}{2} \times 2 = \frac{34}{2} = 17$

$\frac{17}{2} \times 4 = \frac{68}{2} = 34$

$\frac{17}{2} \times 3 = \frac{51}{2} = 25\frac{1}{2}$

$\frac{17}{2} \times 5 = \frac{85}{2} = 42\frac{1}{2}$

119

Having our just Measure of Wine, let us try the Prices how they suit our purpose by Alligation medial, for considerable errours may be caused by misapplica-

application of Prices, when the parts were truly taken; but here-under it is apparent that each parcel multiplied by its price the Sum of the Products is 1428 pence, which divided by 119 give 12 for the common price.

17	34	$25\frac{1}{2}$	$42\frac{1}{2}$
8	14	7	15
136	136	$178\frac{1}{2}$	$217\frac{1}{2}$
	34		42
	476		$637\frac{1}{2}$

136
476
$178\frac{1}{2}$
$637\frac{1}{2}$
1428

119) 1428 (12
119
238
238
0

This Proof by Alligation medial I do not account a needleſs curioſity; but very uſeful to be throughly underſtood, for experience informs me, that young Men being defective in Skill, Care and Memory, are apt to miſtake in ſeveral points, but eſpecially one: That is ſo as to eſteem the ſeveral parcels to be of the ſeveral prices from which the differences (by which they are found) were originally taken, and not (as the truth is) of the prices to whom in the counterchange they were annexed.

As

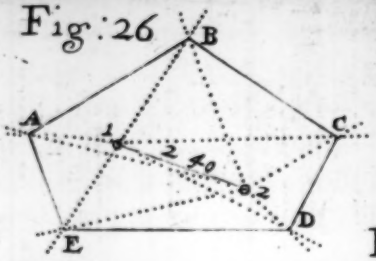
As to that which ſeems to look like an unneceſſary affectation of Novelty, in linking the prices and differences by ſeparate couples; I deſigned at no higher thing then to free the Printer (if this paſs his hand) from the trouble of looking up his duſty Cuts of Semilunes interſecting or encloſing one another, as in our uſual Books of *Arithmetick*. Though the truth is, in ſuch caſes as this before us, no linking at all is needful; but when the common Quotient is multiplied ſeverally by all the differences, any price above the common may be aſſigned to any product made by the difference originally belonging to an under price, and contrarily, ſo as true couples be obſerved. So here I might have aſſigned the price 15 to the parcel 34, found out by the difference 4 originally belonging to 8, and the price 14 to the parcel $42\frac{1}{2}$, found by multiplying the common Quotient by 5 the difference of 7 from the common price: But then I muſt be ſure to do juſtice, ſo as to aſſign the price 8 to the parcel $25\frac{1}{2}$ found out by 3 the difference of 15 from the common price, and the price 7 to the parcel 17 ariſing from 2 the difference of 14 as here is plain.

$17 \times 7 = 119$
$25\frac{1}{2} \times 8 = 204$
$34 \times 15 = 510$
$42\frac{1}{2} \times 14 = 595$
1428

119) 1428 (12
119
238
238
0

FINIS.

Fig: 26



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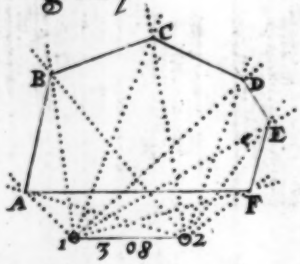

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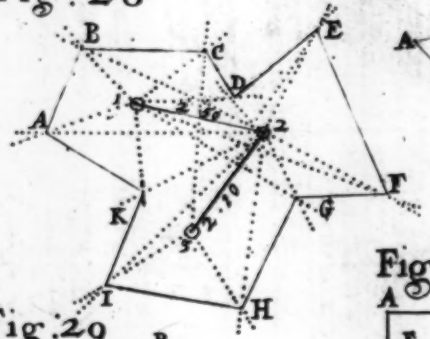

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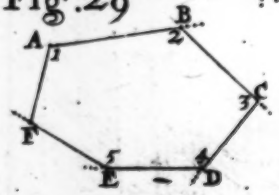

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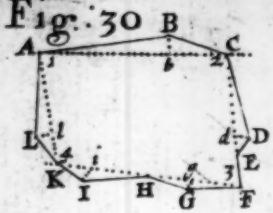

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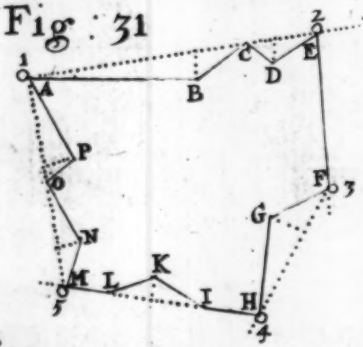

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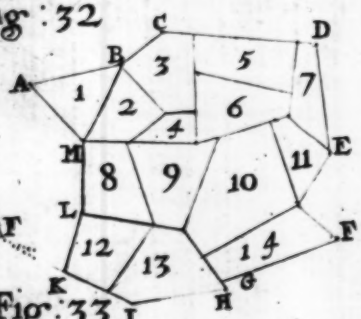

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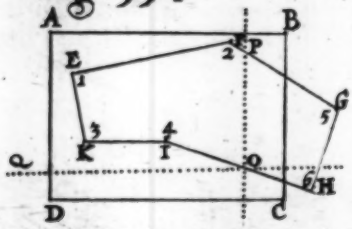

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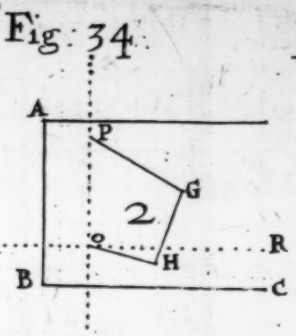
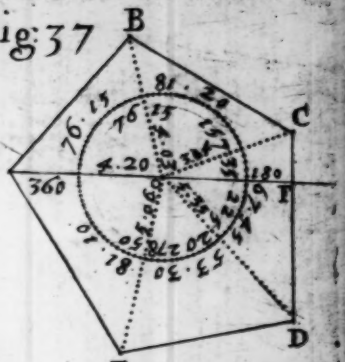


Fig: 35

Fig: 36

Fig: 37



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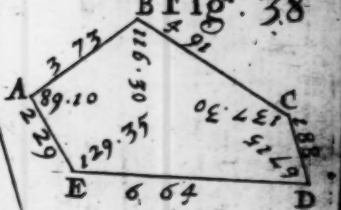

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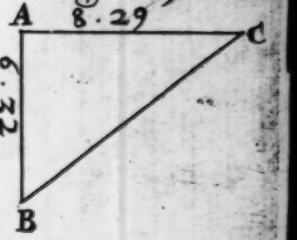

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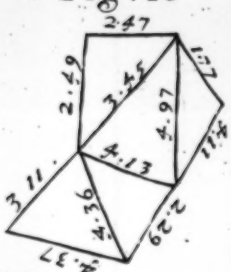

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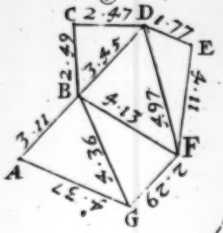

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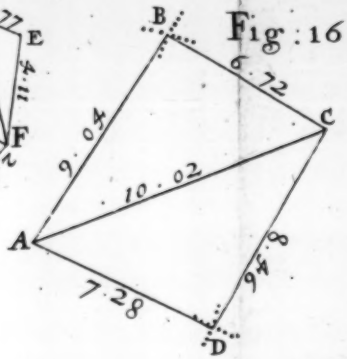

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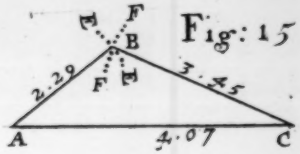

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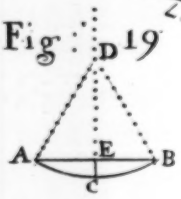

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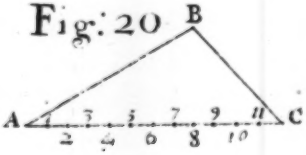
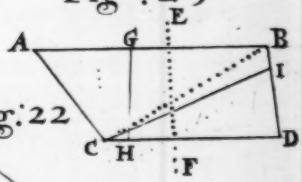


Fig: 23



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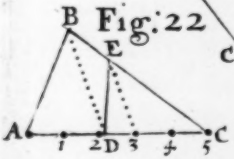

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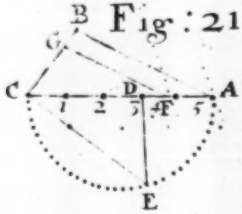

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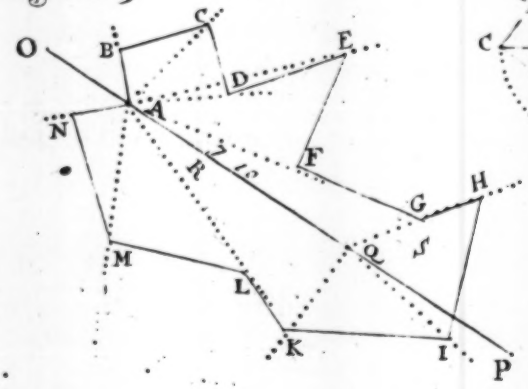

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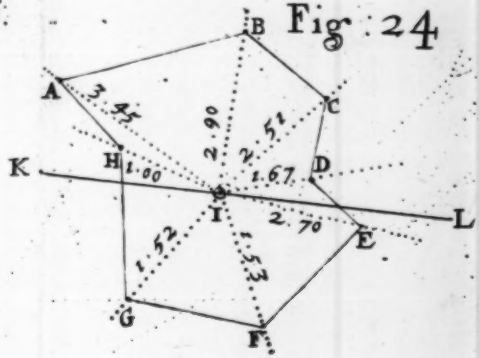
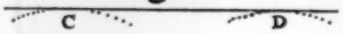


Fig: I.



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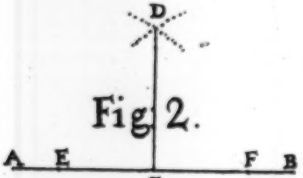

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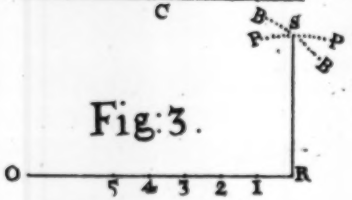

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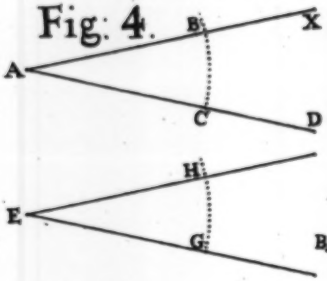

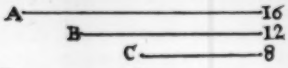

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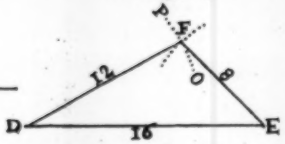

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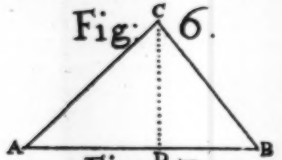

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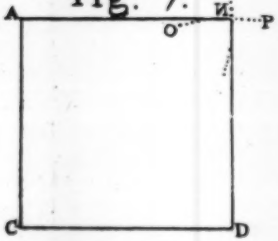

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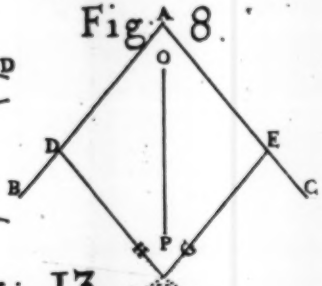

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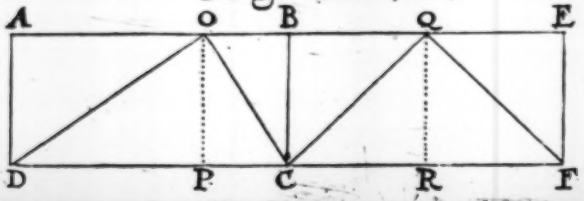
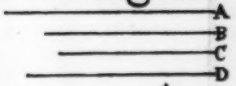


Fig: 9.

Fig: 10.



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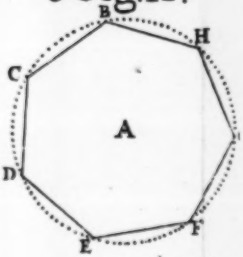

Fig 12

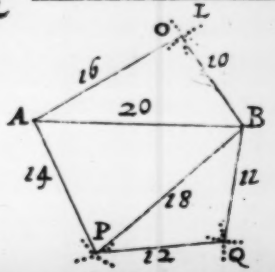

Fig: 14.